Typical Reinforced Concrete Building
Cast-in-place reinforced concrete structures have monolithic slab to beam and beam to column connections. Monolithic comes from the Greek words mono (one) and lithos (stone). Consequences of monolithic construction include:

- beam-to-column connections transfer moment making reinforced concrete frames highly indeterminate, and
- the deck slab and the beam are one piece of concrete, making the beams “T-shaped”

A typical reinforced concrete floor system is shown in the sketches below.

It’s important to have a clear picture in one’s mind of the concrete structure represented by the 2D sketches above. The orthographic views on the following page of the same floor system are intended to help in this regard.
Isometric View of Reinforced Concrete Floor System

Cut-away showing beam dimensions
Placement of Reinforcement
Steel reinforcement is placed in the tension zones of reinforced concrete beams, as indicated by the crack patterns shown below.

Reinforcement in Tensile Zones

Factored Moments due to Dead + Live Loads ($M_u$)
Because concrete frames are highly indeterminate, the moments due to factored dead and live loads are typically calculated with the aid of a computer program. Alternatively, designers often use the American Concrete Institute (ACI) moment coefficients (shown below) which represent the envelope of moments due to dead load plus various live load span load patterns.

ACI Moment Coefficients
Example. Check the flexural strength in the office building indicated below at three locations:

1. a one-foot-wide section of slab
2. the middle of an exterior span of an interior beam
3. at the interior support of an exterior span of an interior beam

Col. Size = 24" x 24"

\begin{align*}
t_{\text{slab}} &= 5 \text{ in} \\
h &= 16 \text{ in} \\
b_{\text{w}} &= 13 \text{ in} \\
f'c &= 4000 \text{ psi} \\
f_y &= 60,000 \text{ psi} \\
\text{unit wt of conc.} &= 150 \text{ pcf} \\
w^{SO} &= 10 \text{ psf} \\
w^{LL} &= 40 \text{ psf}
\end{align*}

Slab Reinforcement:

#4 @ 15"

Beam Reinforcement:

Midspan: 3 #7 bars
Support: 8 #6 bars
#3 stirrups

1. One-foot-wide strip of slab

\begin{align*}
\varepsilon_s &= 0.003 \\
0.85f'c &= \text{Stresses} \\
A_s &= \text{Forces}
\end{align*}
Example: Analysis of Reinforced Concrete Floor

Calc. $M_u$:

Super-imposed Dead Load
\[ w_{SDL} = (SDL)(\text{tributary width}) = (0.010\text{ksf})(1\text{ft}) = 0.010\text{kf} \]

Self-weight
\[ w^w = \text{(unit wt) (width)(depth)} = (0.150\text{kcf})(1\text{ft})(5\text{in}/12\text{in/ft}) = 0.0625\text{kf} \]

Live load
\[ w^L = (LL)(\text{tributary width}) = (0.040\text{kost})(1\text{ft}) = 0.040\text{kf} \]

Factored load
\[ w_u = 1.2(w^w + w_{SDL}) + 1.6(w^L) = 1.2(0.0625\text{kf} + 0.010\text{kf}) + 1.6(0.040\text{kf}) = 0.151\text{kf} \]

Moment due to factored loads
\[ M_u = (1/M_{coef})w_u(L_n)^2 \]
\[ M_{coef} = 10, \text{ max. moment for continuous beam, [FE Ref. pg. 144]} \]

Clear span
\[ L_n = \text{beam spacing} – \text{width of beam web} = 12\text{ft} – 13\text{in}/12\text{in/ft} = 10.92\text{ft} \]
\[ M_u = (1/10)(0.151\text{kf})(10.92\text{ft})^2, \]
\[ M_u = 1.80\text{kft} \]

Calc. $\phi M_u$:

Calc. depth of stress block
from $\Sigma F_{ht}$: $C = T \rightarrow 0.85 f_c a b = A_s f_y$ (Assume steel yields)

Calculate the area of steel in a one-foot wide strip of slab:
\[ A_s \text{ of one #4 bar} = 0.20 \text{ in}^2 \text{ [FE Ref. pg 144]} \]
\[ A_s \text{ in a 1}^{\text{ft}} – \text{wide strip} = (0.20 \text{ in}^2/\text{bar})(\frac{1\text{bar}}{15\text{in}})(\frac{12\text{in}}{1^{\text{ft}} – \text{wide strip}}) = 0.160 \text{ in}^2/\text{1 - ft strip} \]
\[ 0.85 (4\text{ksi}) a (12\text{in}) = (0.160 \text{ in}^2)(60\text{ksi}), \quad a = 0.235\text{m} \]

Calc. effective depth, $d$
\[ \text{cover} = 0.75\text{in}, \text{ [ACI Clear Cover Reqt's: Concrete not exposed to weather, Slabs, No 11 bar and smaller]} \]
\[ d = h – \text{cover} – \phi_{bar}/2 = 5\text{in} – 0.75\text{in} – 0.50\text{in}/2, \quad d = 4.00\text{in} \]

Calc. strain in steel, $\varepsilon_s$
\[ \frac{0.003}{a/\beta_1} = \frac{\varepsilon_s + 0.003}{d} \text{ [FE Ref. pg. 145]} \]
\[ \beta_1 = 0.85 \text{ for } f'_c = 4\text{ksi} \text{ [FE Ref. pg. 144]} \]
\[ \frac{0.003}{0.235\text{in}/0.85} = \frac{\varepsilon_s + 0.003}{4.00\text{in}} \]
\[ \varepsilon_s = 0.0404 \]

- $\varepsilon_s = 0.002$, so steel yields as assumed
- $\varepsilon_s = 0.004$ = min. for beams [FE Ref. pg. 145]
- $\varepsilon_s = 0.90$ since $\varepsilon_s > 0.005$ [FE Ref. pg. 145]
Calc. $\phi M_n$

\[ \phi M_n = \phi A_s f_y (d - a/2) = (0.90)(0.160 \text{ in}^2)(60 \text{ ksi})(4.00 \text{ in} - 0.235 \text{ in}/2) / 12 \text{ in}/\text{ft} \]

\[ \phi M_n = 2.80 \text{ k-ft} \]

\[ \phi M_n = 2.80 \text{ k-ft} > 1.80 \text{ k-ft} = M_w, \text{ OK} \]

2. Midspan of exterior span: T-beam

Calc. $M_u$

Self-weight of T-beam: The self-weight of the T-beam is equal to the unit weight of reinforced concrete (typically 150 pcf) times the cross-section area of the T-beam. The flange of the T-beam extends halfway to the nearest beam on either side, which is typically equal to the beam spacing.

\[ w^{sw} = \text{unit}_\text{wt} \times A_{T\text{-beam}} \]

\[ A_{T\text{-beam}} = \text{the area of the slab} + \text{the area of the beam web that extends below the slab} \]

\[ A_{T\text{-beam}} = (\text{beam spacing} \times t_{\text{slab}}) + (h - t_{\text{slab}}) \times b_w \]

\[ w^{sw} = (0.150 \text{kpcf})[12 \text{ ft} \times (5\text{"}/12\text{ in/ft}) + (16\text{ in} - 5\text{ in}) \times 13\text{ in} \times (1 \text{ ft}^2/144 \text{ in}^2)] = 0.899 \text{ klf} \]

\[ w^{SD} = (w^{SD})(\text{beam spacing}) = (0.010 \text{ksf})(12') = 0.120 \text{ klf} \]

\[ w^L = (w^L)(\text{beam spacing}) = (0.040 \text{ksf})(12') = 0.480 \text{ klf} \]

\[ w_u = 1.2 (w^{sw} + w^{SD}) + 1.6 w^L \]

\[ w_u = 1.2 (0.899 \text{ klf} + 0.120 \text{ klf}) + 1.6 (0.480 \text{ klf}) = 1.991 \text{ klf} \]

\[ M_u = w_u L_n^2 / 14 \quad \text{(moment coefficient for exterior span, midspan)} \]

\[ L_n = \text{clear span} = \text{column spacing} - \text{width of column} = 29\text{ ft} - 2\text{ ft} = 27\text{ ft} \]

\[ M_u = (1.991 \text{ klf}) (27 \text{ ft})^2 / 14 \]

\[ M_u = 103.7 \text{ k-ft} \]
Calc. $\phi M_n$:

**effective flange width, $b_f$**: [FE Reference, pg 146]

\[
b_f = \text{minimum of: } \frac{\text{clear span}}{4} \quad 16 \cdot \text{t. slab} + b_w \quad \text{beam spacing}
\]

\[
27'(12")/4 = 81" \quad 16(5") + 13" = 93" \quad 12' \times 12''/" = 144"
\]

\[
b_f = 81"
\]

Calc. stress resultants

Assume the compression block does not extend below the bottom of the flange (typical).

$\Sigma F_H: C = T$

\[
0.85 f'c a b_f = As \quad \text{Assume steel yields}
\]

\[
As = 3(0.60in^2) = 1.80in^2
\]

\[
0.85 \times 4(ksi) \times a (81") = (1.80 \times \text{in}^2) \times (60 \text{ksi}), \quad a = 0.392"
\]

\[
a = 0.392" < 5" = t_{\text{slab}}
\]

therefore compression block does not extend below flange, as assumed

Note: if $a$ had been $> t$, then

$\Sigma F_H: C = T$

\[
0.85 f'c [(b_f - b_w) t + b_w a] = As \quad \text{Assume steel yields}
\]

Solve equation above for $a$, and proceed as below, except:

when calculating $\phi M_n$, add the moment due to the compression in the flange

\[
\phi M_n = \phi [As fy (d - a/2) + C_t (a/2 - t/2)], \quad \text{where} \quad C_t = 0.85 f'c t (b_f - b_w)
\]
Calc. strain in steel
cover = 1.5", [ACI Clear Cover Reqt’s: Concrete not exposed to weather, Beams & columns]

\[
\frac{0.003}{c} = \frac{0.003 + \varepsilon_s}{d}
\]

\[
d = 16" - [1.5" + \frac{3"}{8} + \frac{1}{2} 7"] = 13.69"
\]

\[
a = \beta(c), \ \beta_i = 0.85 for f'c = 4 ksi
\]

\[
0.392" = 0.85 c, \ \ c = 0.461"
\]

\[
\frac{0.003}{0.461"} = \frac{0.003 + \varepsilon_s}{13.69"}, \ \ \varepsilon_s = 0.0861
\]

\[
\therefore (1) \ \text{steel has yeilded } (\varepsilon_s > \varepsilon_y = 0.002), \ \text{as assumed}
\]

(2) \(\varepsilon_s > 0.004 \ \text{(ACI min for beams), OK}\)

(3) \(\phi = 0.90 \ \text{(\varepsilon_s >= 0.005)}\)

\[
\phi M_n = \phi [As f_y (d - a/2)]
\]

\[
\phi M_n = 0.90 [(1.80in^2)(60 ksi) (13.69" - 0.392")/2)] (1' / 12"),
\]

\[\phi M_n = 109^{k-ft} > 104^{k-ft} = M_u, \ \text{OK}\]

3. At Interior Side of Interior Support

The bottom of the web is in compression, everything above the neutral axis is cracked. The top steel is spread laterally across the flange (therefore it is always in one layer).

Calc. \(M_u\)

\[M_u = w_u L_n^2 / 10 \ \text{(moment coefficient for exterior span, interior support)}\]

\[= (1.991 \text{ klf}) (27 \text{ ft})^2 / 11\]

\[M_u = 145.1 \text{ k-ft}\]
Calc. $M_n$

Calc. depth of stress block

$\Sigma f_H = C = T$

$0.85 f'c a b_w = A_s f_y \quad \text{(Assume steel yields)}$

$A_s = (8 \text{ bars})(0.44 \text{ in}^2 \text{ per bar}) = 3.52 \text{ in}^2$

$0.85 (4 \text{ ksi}) a (13") = (3.52 \text{ in}^2) (60 \text{ ksi}), \quad a = 4.778"$

Calc. strain in steel.

$$\frac{0.003}{c} = \frac{0.003 + \varepsilon_s}{d}$$

$$d = 16" - [1.5" + \frac{3"}{8} + \frac{1 6"}{2 8}] = 13.75"$$

$$a = \beta_1 c, \quad \beta_1 = 0.85 \text{ for } f'_c = 4 \text{ ksi}$$

$4.778" = 0.85 c, \quad c = 5.621"$

$$\frac{0.003}{5.621"} = \frac{0.003 + \varepsilon_s}{13.75"}, \quad \varepsilon_s = 0.00434$$

∴ steel has yeilded ($\varepsilon_s > \varepsilon_y = 0.002$), as assumed

&

$$\phi = 0.48 + 83(0.00434) = 0.840$$

&

$$\varepsilon_s > 0.004 \quad (OK)$$

Calc. $\phi M_n$

$$\phi M_n = \phi [A_s f_y (d - a/2)]$$

$$\phi M_n = 0.840 [(3.52 \text{in}^2)(60 \text{ ksi}) (13.75" - 4.778"/2)] (1' / 12"),$$

$$\phi M_n = 168^{k-ft} > 145^{k-ft} = M_w, \quad \text{OK}$$