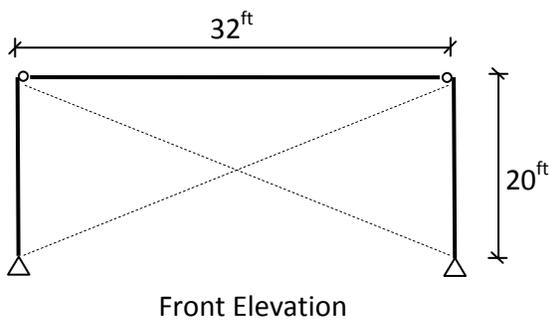
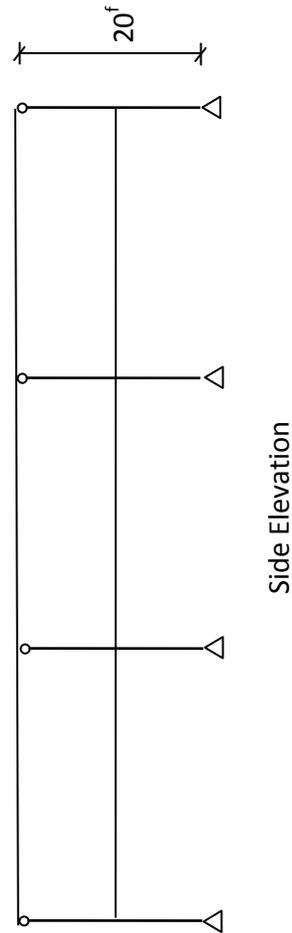
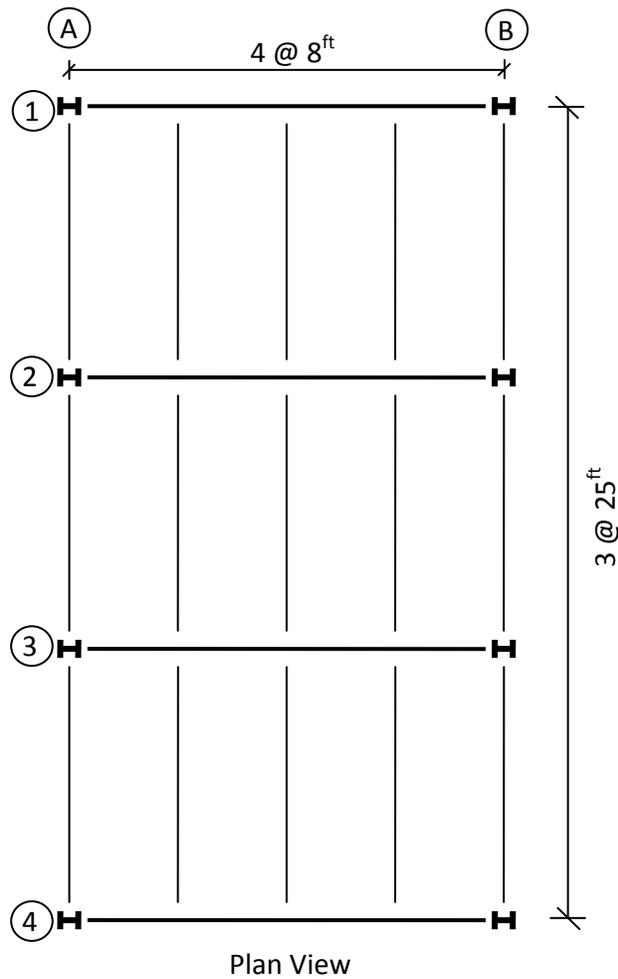


Check the strength of each type of member in the one-story steel-frame building below.



$F_y = 50^{\text{ksi}}$ all members
 $F_u = 65^{\text{ksi}}$

	Shape
Purlins	Z12x40
Girders	W21x44
Columns	W16x36
X-Bracing	L2x2x1/8 $A_g = 0.484 \text{ in}^2$

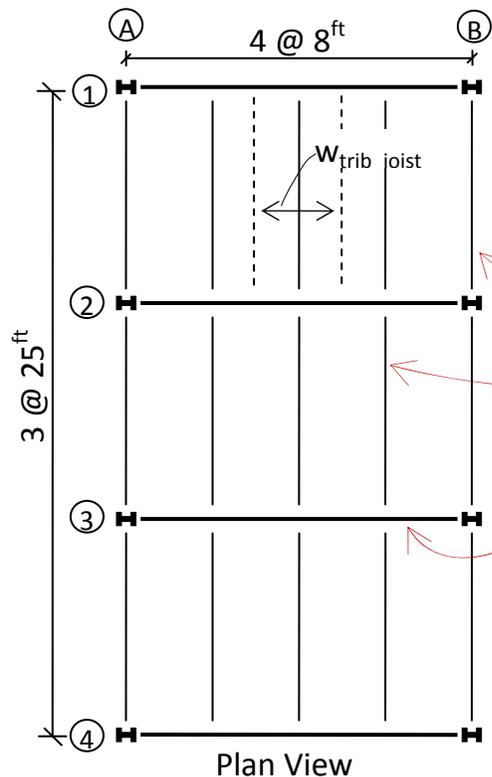
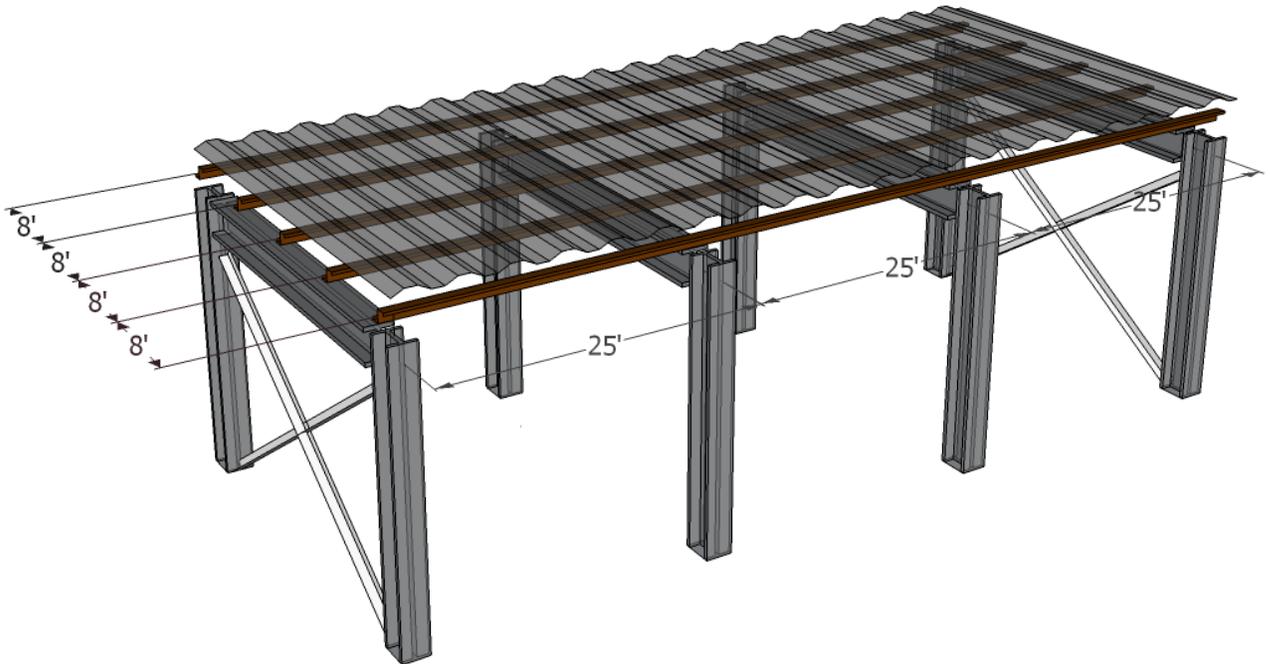
Loads:

- 3.5" thick light-weight concrete slab (unit weight = 120 pcf)
- LL = 40 psf
- WL = 30 psf

Load Combinations:

- 1.2D + 1.6L
- 1.2D + 1.6L

Identify joists and girders. The roof deck is supported by the Z-shaped purlins, which run transverse to the deck corrugations (see “exploded” building in figure below). The purlins are in turn supported by the girders, which run transverse to the purlins. The girders are supported at their ends by the columns.



First of all, the “H”-shaped symbols represent the columns.

In the Plan View at left, we see that some of the “vertical” members in the sketch are attached directly to columns, but that some of the “vertical” members are attached at their ends to other beams.

The ends of the “horizontal” members, on the other hand, all attach to columns.

Therefore, the horizontal members in the sketch are girders, and the vertical members are purlins (if supporting a roof) or joists (if supporting a floor).

Purlin or Joist –max M_u

Purlins or joists are frequently continuous beams over several spans (in this example, the purlins are continuous over three 25-foot spans). Loading all of the spans of a continuous beam may not cause the maximum bending moment. Although the position of dead load is given, the position of live load is variable and the structural engineer must determine the loading causing the maximum bending moment.

One way of determining the loading causing the maximum bending moment is to apply all possible load configurations, one at a time, and select the loading causing the maximum effect. In this class, we will assume that the location of the maximum bending moment due to dead plus live loads is the location with the maximum bending moment due to dead loads.

$$\text{Location of max } M^{D+L} = \text{Location of max } M^D$$

This is the case for continuous beams with equal span lengths. T

Our procedure for calculating the maximum moment due to factored loads will be:

1. Apply the dead load to all spans and calculate the moment (M^D) using charts from the AISC manual
2. Assume that the location of the max M^{D+L} = the location of the max M^D . Draw the influence diagram for moment for this location.
3. Apply the live load to the spans indicated by the influence diagram and calculate the moment (M^L) using the AISC charts.
4. Calculate M_u from $1.2 M^D + 1.6 M^L$.

Example.

Dead Loads:

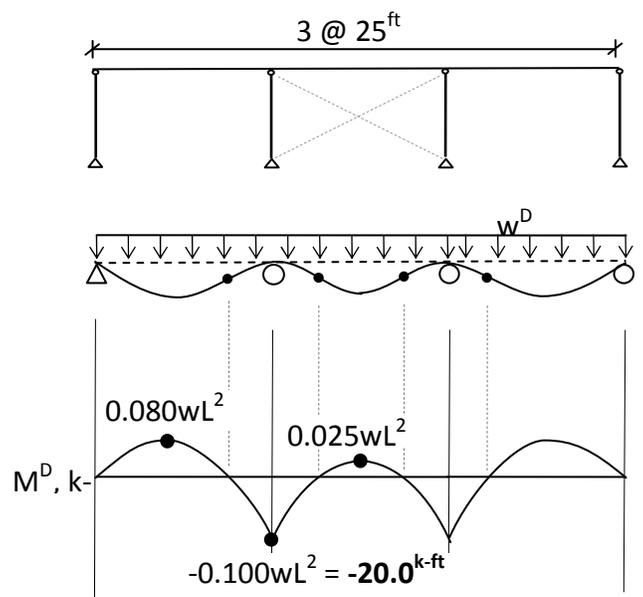
Weight of slab = $3.5''/12'' \times 120\text{pcf} = 35\text{ psf}$
width of slab supported by joist = tributary width
 $w_{\text{trib}} = 8\text{ft}$ (see sketch on bottom of Pg. 2)
uniform load due to weight of slab = w^{slab}
 $w^{\text{slab}} = 35\text{psf} \times 8\text{ft} = 0.280\text{klf}$

self-weight of joist = 40plf
uniform load due to self weight of joist = $w^{\text{sw}} = 40\text{plf}$

uniform load due to all dead loads = w^D
 $w^D = 0.280\text{klf} + 40\text{plf}/1000\text{lb/k} = 0.320\text{klf}$

Max $M^D = 0.100 wL^2$ from AISC charts
 $M^D = 0.100 (0.320\text{klf})(25\text{ft})^2$

$$\underline{M^D = 20.0\text{k-ft}}$$



Live Loads:

Assume A_T = area supported by one span of joist (conservative)

$$LL_{reduction} = \left(0.25 + \frac{15}{\sqrt{k_{LL} A_T}} \right), \quad 0.4 \leq LL_{reduction} \leq 1.0$$

$$k_{LL} = 2 \text{ (beams)}$$

$$A_T = \text{tributary area of joist} = (8^{ft})(25^{ft}) = 200^{sf}$$

$$LL_{reduction} = \left(0.25 + \frac{15}{\sqrt{(2)(200^{sf})}} \right) = 1.00$$

Therefore reduced live load = $L_{red} = 40 \text{ psf} \times 1.0 = 40 \text{ psf}$

$$w^{Lred} = (40.0^{psf})(8^{ft}) / (1000^{lb/k}) = 0.320^{klf}$$

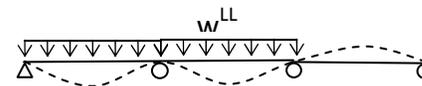
↙ tributary width for joist (see sketch on bottom of Page 2)

Assume max M^{D+L} occurs at location of max M^D

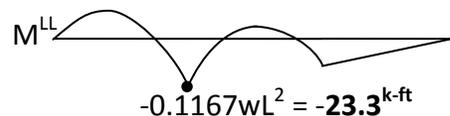
Influence Diagram for M at Support 2:



Span loading to cause max. -M at Support 2:



$$\max M^L = -0.1167(0.320^{klf})(25^{ft})^2 = -23.3^{k-ft}$$



max $M_u =$

$$1.2(-20.0^{k-ft}) + 1.6(-23.3^{k-ft})$$

$$\max M_u = -61.3^{k-ft}$$

Joist –unity check

$L_{b_comp_flange} = 0 < L_p$ (assume compression flange is braced full length by roof diaphragm).

Therefore, calculate $\phi M_n = \phi M_p$

$$\phi M_p = 214^{k-ft}, \text{ [AISC Table 3-2, pg 154 FE Ref.]}$$

$$U.C. = \frac{M_u}{\phi M_n} = \frac{61.3^{k-ft}}{214^{k-ft}} = 0.29 < 1.0, \text{ OK}$$

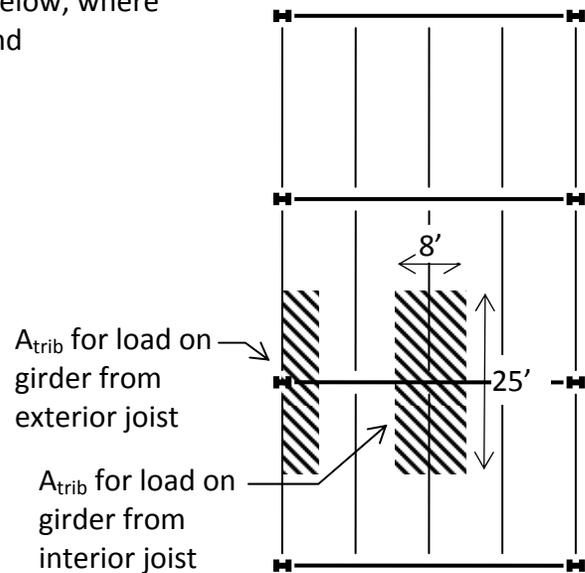
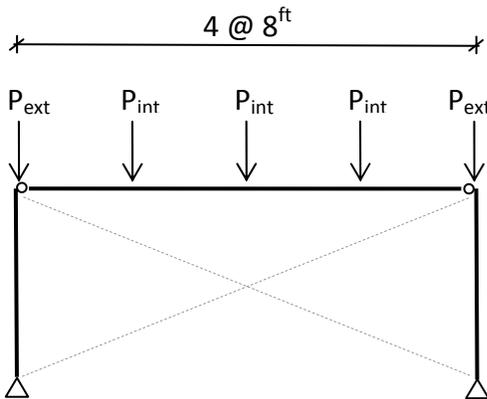
Girder –max M_u

Since all girders are the same size, the girder with the largest unity check will be the girder with the largest loads. Therefore, analyze a girder from an interior frame.

The loads on this girder are indicated in the sketch below, where

P_{ext} = the loads on the girder from exterior joists and

P_{int} = the loads on the girder from interior joists



The tributary areas used to calculate these loads are shown in the sketch at right.

$$w_{t_{slab}} = 35^{psf}$$

$$w_{t_{joists}} = 40^{plf}$$

$$w_{t_{girder}} = 44^{plf}$$

Concentrated load due to dead load at an interior joists = P_{int}^D

$$P_{int}^D = (35^{psf})(25^{ft})(8^{ft}) + (40^{plf})(25^{ft}) + (44^{plf})(8^{ft}) = 8.35^k$$

Since the girder is a single span, there is no need to consider span load patterns for live load.

For live-load reduction, tributary area (A_t) equals tributary width (25') times span length (32')

$$A_t = 25^{ft} \times 32^{ft} = 800^{sf}$$

$$LL_{reduction} = 0.25 + \frac{15}{\sqrt{(2)(800^{sf})}} = 0.625 \text{ (between 0.4 and 1.0, OK)}$$

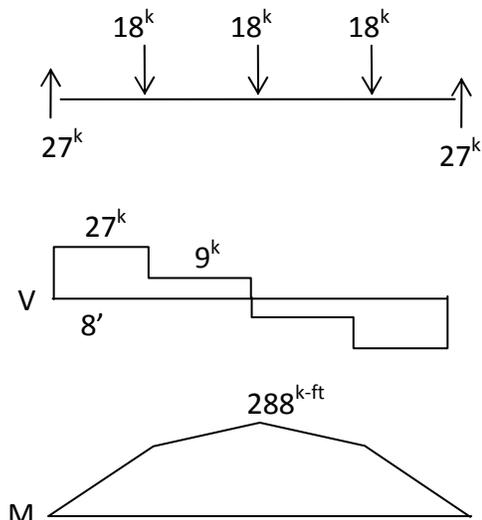
0.4 and 1.0, OK

$$P_{int}^L = (40^{psf})(0.625)(25^{ft})(8^{ft}) = 5.0^k$$

$$P_{u_{int}} = 1.2(8.35^k) + 1.6(5.0^k) = 18.0^k$$

After drawing the shear and moment diagrams (at right),

$M_u = 288^{k-ft}$



Girder –unity check

$$L_p = 4.45^{\text{ft}}, L_r = 13.0^{\text{ft}} \quad [\text{AISC Table 3-2, pg 154 FE Ref.}]$$

$$L_p < (L_b = 8^{\text{ft}}) < L_r$$

$$\therefore \phi M_n = C_b [\phi M_p - BF(L_b - L_p)] < \phi M_p$$

$$C_b = 1.0 \text{ (always for this class)}$$

$$\phi M_p = 358^{\text{k-ft}} \quad [\text{AISC Table 3-2, pg 154 FE Ref.}]$$

$$BF = 16.8^{\text{k}} \quad [\text{AISC Table 3-2, pg 154 FE Ref.}]$$

$$\phi M_n = (1) [358^{\text{k-ft}} - 16.8^{\text{k}}(8^{\text{ft}} - 4.45^{\text{ft}})] \quad [\text{pg 150, FE Ref.}]$$

$$\phi M_n = 298^{\text{k-ft}} (< 358^{\text{k-ft}} = \phi M_p)$$

$$UC = \frac{M_u}{\phi M_n} = \frac{288^{\text{k-ft}}}{298^{\text{k-ft}}} = 0.97 < 1, \text{ OK}$$

Column –max P_u (critical column = interior column)

$$SDL = 35^{\text{psf}}$$

$$wt_{\text{joists}} = 40^{\text{plf}}$$

$$wt_{\text{girder}} = 44^{\text{plf}}$$

$$P^D = (35^{\text{psf}})(25^{\text{ft}})(16^{\text{ft}}) + (40^{\text{plf}})(25^{\text{ft}})(5^{\text{joists}}/2) + (44^{\text{plf}})(16^{\text{ft}})$$

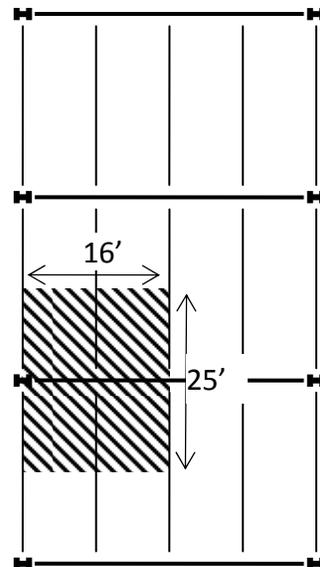
$$P^D = 17.2^{\text{k}}$$

$$A_t = (32'/2)(25') = 400^{\text{sf}}$$

$$LL_{\text{reduction}} = \left(0.25 + \frac{15}{\sqrt{(4)(400^{\text{sf}})}} \right) = 0.625$$

$$P^L = (40^{\text{psf}})(0.625)(400^{\text{sf}}) = 10.0^{\text{k}}$$

$$P_u = 1.2(17.2^{\text{k}}) + 1.6(10.0^{\text{k}}) = 36.6^{\text{k}}$$

**Column –unity check**

$$A = 10.6 \text{ in}^2, r_x = 6.51 \text{ in}, r_y = 1.52 \text{ in}, \quad [\text{Table 1-1, pg 153 FE Refer}]$$

$$\frac{k_x L_{u-x}}{r_x} = \frac{(1.0)(20^{\text{ft}} \frac{12^{\text{in}}}{1^{\text{ft}}})}{6.51 \text{ in}} = 36.9$$

$$\frac{k_y L_{u-y}}{r_y} = \frac{(1.0)(10^{\text{ft}} \frac{12^{\text{in}}}{1^{\text{ft}}})}{1.52 \text{ in}} = 78.9 \quad \leftarrow \text{controls}$$

$$KL/r = 79 \text{ (round up)}$$

$$\phi F_{cr} = 28.5^{\text{ksi}} \quad [\text{AISC Table 4-22, pg 157 FE Ref.}]$$

$$\phi P_n = \phi F_{cr} A = (28.5^{\text{ksi}})(10.6 \text{ in}^2) = 302^{\text{k}}$$

$$UC = \frac{P_u}{\phi P_n} = \frac{36.6^{\text{k}}}{302^{\text{k}}} = 0.12 < 1.0, \text{ OK (but over-designed)}$$

End Wall Cross-Bracing – max T_u due to Wind Loads

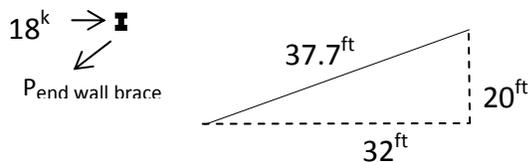
Half of wind load on long wall of building goes down to ground, and half goes up to the roof diaphragm. The wind load to the roof is resisted by x-bracing in the end walls (40-foot-wide walls). Therefore the load to one end wall is

$$P_{U \text{ end wall}}^W = (1.6)(WL)\left(\frac{h_{col}}{2}\right)\left(\frac{\text{Length of Bldg}}{2}\right) = (1.6)(30 \text{ psf})\left(\frac{20'}{2}\right)\left(\frac{75'}{2}\right) = 18.0^k$$

End-wall X-bracing:

$$T_{\text{end-wall brace}} = (18^k)(37.7^{\text{ft}} / 32^{\text{ft}})$$

$$T_{U \text{ end-wall brace}} = 21.2^k$$



End Wall Cross-Bracing – ϕT_n

$$A_g = 0.484 \text{ in}^2 \text{ [Angles Properties]}$$

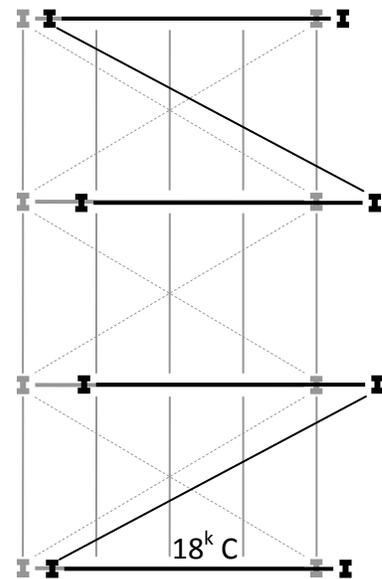
$$\phi A_g F_y = 0.9 (0.484 \text{ in}^2) (50 \text{ ksi}) = 21.8 \text{ k} \quad \leftarrow \text{controls}$$

$$\phi A_e F_u = 0.75 (1.0 \times 0.484 \text{ in}^2) (65 \text{ ksi}) = 23.6 \text{ k}$$

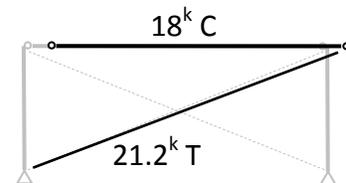
(assume $U = 1.0$ for this class)

$$\phi T_n = 21.8 \text{ k}$$

$$UC = \frac{T_u}{\phi T_n} = \frac{21.2^k}{21.8^k} = 0.97 < 1.0, \text{ OK}$$



Plan View



Elevation View