The normal procedure in designing a structural member is to first calculate the loads on the structure, then calculate the internal forces on the particular member, and finally check if the internal forces will cause the member to fail (see Figure 1). If our calculations indicate a member will fail, then we change the structure in some way and repeat the process.

**Figure 1. Structural Design Process.**

We followed the structural design process in our first project: Design, Build and Test a Bridge of Plastic Straws. The process was considerably simplified for two reasons:

1. we assumed only one failure mode (axial buckling), and
2. all straws had the same cross-section.

**Material Properties and Cross-Section Properties**

In our next project (Design, Build and Test a Bridge of Basswood), we will consider five failure modes and we will use basswood pieces with many different cross-sections. We will see in the next section that each failure mode depends on a cross-section property and a material property. But cross-section properties can be calculated from cross-section dimensions which can be measured. We will therefore use our test results for a particular failure mode to calculate the material property associated with that failure mode.

For example, the axial load to cause a plastic straw to buckle ($P_{cr}$) depends on the modulus of elasticity ($E$) and the moment of inertia ($I$).

$$P_{cr} = \frac{\pi^2 EI}{L_u^2}$$

The moment of inertia is a cross-section property and the modulus of elasticity is a material property. If we had used plastic straws of different diameters in Project 1, we would have measured the diameters and wall thicknesses of each type of straw and then calculated the moment of inertia, $I$. Knowing $I$, $L_u$ and $P_{cr}$ for a particular straw, we could then calculate its modulus of elasticity ($E$). We would then use $E$, a material property, to predict the buckling loads for straws with any combination of cross section and unbraced length.
Failure Modes

1. Axial Failure
   1.1. Tensile Failure
       - fibers rupture

   1.2. Compression Failure
       - fibers crush

   1.3. Axial Buckling Failure

2. Bending Failure

3. Shear Failure

Stress
The failure modes described above can be classified as either material failures (tensile, compression, bending and shear failures) or stability failure (buckling failure). Material failures occur on the microscopic level: in wood, the fibers rupture, crush or slide past each other; in steel, atoms in the crystal lattice slide past one another. The microscopic forces causing these material failures are characterized by the force intensity, calculated as the force per unit area or stress.

We will consider two types of stress:

a) normal stress ($\sigma$), in which the force is applied perpendicular or normal to the cross section, and

b) shear stress ($\tau$), in which the force is applied parallel to the cross section.

We predict a member will fail when structural loads cause a stress that exceeds the failure stress. Failure stresses can be considered to be material properties. Building codes specify...
maximum allowable stresses (allowable stress = failure stress / factor of safety) for common material types. Typical allowable stresses for each of four failure modes are shown for steel and wood in Table 1 below. The following convention (common in steel and wood design) is used in Table 1:

- stresses due to loads = $f$, allowable stresses = $F$
- tensile, axial compressive, bending and shear stresses use subscript “t”, “a”, “b” and “v”, respectively.

The part of the design process labeled “Check for Failure” in Figure 1 involves ensuring the stresses due to loads are less than the allowable stresses, for each failure mode due to material failure. That is:

$$f_a < F_t$$
$$f_a < F_a$$
$$f_b < F_b$$
$$f_v < F_v$$

Typically, the allowable stresses above are modified (decreased) to account for stability failure. For example, the allowable compressive stress, $F_c$, is decreased if the member will fail by axial buckling before failing by compressive material failure. For this class, the allowable stresses will either be given to you (for hand-calculation problems) or will be determined by RISA based on the material type and unbraced lengths you input for each member. The stresses due to loads ($f_t$, $f_c$, $f_b$, and $f_v$) will be calculated by you (for hand-calculation problems) or be determined by RISA.
The stresses due to loads are calculated directly from the internal member forces:
- axial force, P
- shear force, V
- bending moment, M

Equations for calculating the stresses from the internal member forces are reviewed in the next section.

**Stress Equations**

1. Normal Stresses

1.1. Normal Stresses due to Axial Forces

1.1.1. Axial Stress, \( f_a \)

Based on the deformed shape (shown above), the strain is constant over the cross-section, the strain magnitude = \( \varepsilon = \frac{\Delta}{L} \) stress, \( \sigma = \varepsilon E \), so the stress is constant over the cross-section also. To relate the magnitude of the stress to the load \( P \)

\[ P = \int \sigma \, dA = \sigma A \] since the stress is constant over the cross-section. Therefore

\[ \sigma = \frac{P}{A} \] or, using RISA notation,

\[ f_a = \frac{P}{A} \]

1.1.2. Allowable Axial Buckling Stress, \( F_cr \)

The Euler Buckling Equation is: \( P_{cr} = \frac{\pi^2 E I}{L^2_u} \). We need to express the Euler buckling load, \( P_{cr} \), in terms of a stress to that it can be compared with the axial compressive stress to fail (crush) the material.

\[ F_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E I}{L^2_u A} = \frac{\pi^2 E r^2}{L^2_u} \] where \( r^2 = \frac{I}{A} \), \( r \) = called the radius of gyration.
Now: \[ F_{cr} = \frac{\pi^2 E}{L_u^2} \text{, and } \frac{L_u}{r} \] is called the slenderness ratio. The higher a member’s slenderness ratio, the lower its \( F_{cr} \). Or in other words, the more slender a member is the more susceptible it is to buckling.

### 1.1.3. Allowable Axial Compressive Stress, \( F_a \)

The allowable compressive stress (\( F_a \)) is the smaller of the allowable stress for material failure (\( F_c \)) or the allowable stress for axial buckling (\( F_{cr} \)).

\[
F_a = \min\{ F_c, F_{cr} \}
\]

For design, \( F_c \) and \( F_{cr} \) do not represent actual failure stresses, but failure stresses divided by a factor of safety. For example, the allowable axial stress for elastic buckling for steel design is:

\[
F_{cr} = \frac{12\pi^2 E}{23\left(\frac{L_u}{r}\right)^2}
\]

### 1.2. Normal Stresses due to Bending Moments, \( f_b \)
Based on the deformed shape (shown above), the strain varies linearly over the cross-section. The strain is zero at the neutral axis of the cross section and a maximum at the extreme “fibers”.

The strain magnitude at distance “y” from the neutral axis (NA) is

\[ \varepsilon = \frac{\Delta}{L} \]
\[ \Delta = \theta y, \]
\[ \varepsilon = \frac{\theta y}{L} \]

but \( \frac{\theta}{L} = K \), where \( K \) = curvature = angle change per unit length

therefore: \[ \varepsilon = K y \]

And stress \( \sigma(y) = \varepsilon(y) E \), so the stress varies linearly over the cross-section also.

To relate the magnitude of the stress to the load \( M \):

\[ M = \int \sigma y \, dA \]
\[ \sigma = \varepsilon E \]
\[ \varepsilon = Ky \]

\[ M = \int KyEydA = KE \int y^2 \, dA = KEI \]

\[ K = \frac{M}{EI} \]

but,

\[ K = \frac{\varepsilon}{y} \]
\[ \varepsilon = \frac{\sigma}{E} \]
\[ K = \frac{\sigma}{Ey} \]
\[ \frac{M}{EI} = \frac{\sigma}{Ey} \]

\[ \sigma = \frac{My}{I}, \] or, using RISA notation, \[ f_b = \frac{My}{I} \]
1.3. Normal Stresses due to Combined Axial and Bending: Unity Check Equation

The steel and wood design codes specify that the combination of normal stresses due to axial and bending internal forces satisfy the “unity check” equations below:

\[
\frac{f_a}{F_a} + \frac{f_{b,c}}{F_b} < 1.0
\]

\[
\frac{f_a}{F_t} + \frac{f_{b,t}}{F_b} < 1.0
\]

\(f_{b,c}\) represents the maximum normal compressive stress due to bending (for RISA: \(f_c\))

\(f_{b,t}\) represents the maximum normal tensile stress due to bending (for RISA: \(f_t\))

2. Shear Stress (\(f_v\)).