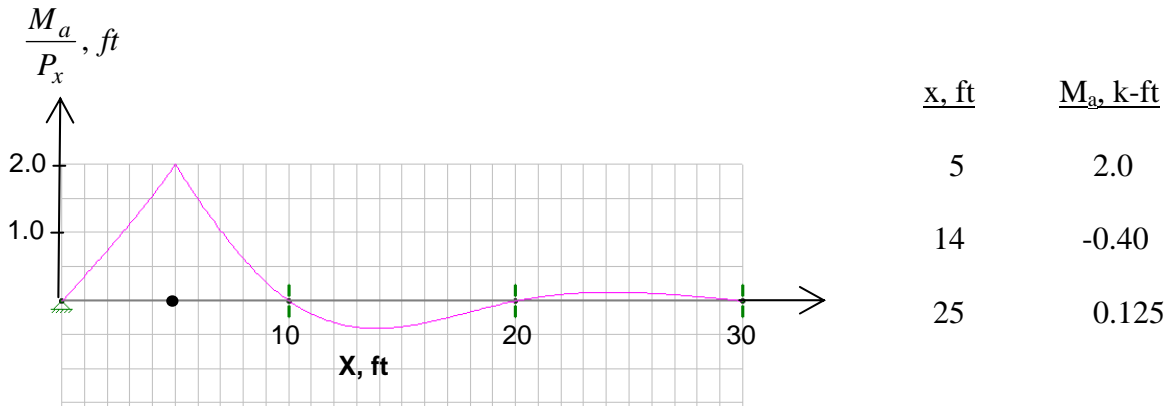
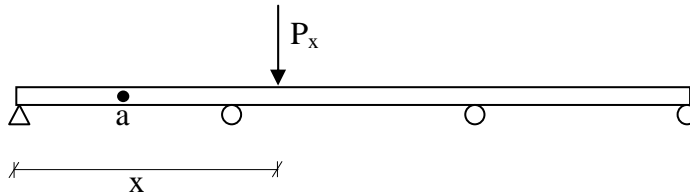


First of all, what is an influence line? An influence line is a plot of a particular member force at a particular location (say moment at the middle of Span 1) due to a moving load,  $P(x)$ .

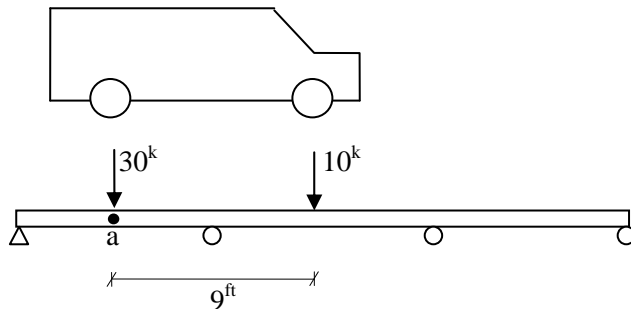


**Influence line** for moment at “a” ( $M_a$ ) due to unit load at “x”,  $P_x$

Influence lines are useful for calculating member forces due to moving loads, for example on crane rails or bridges. A critical location is first identified (at midspan say) and the influence line is constructed by calculating the moment at midspan due to a unit point load at each location ( $x$ ) along the bridge. The influence line can then be used to:

- identify the location of the point load to cause max. moment at midspan
- calculate the value of this moment
- calculate the moment at midspan due to several point loads

Example: Calculate the  $M_a$  due to the truck axle loads shown below.



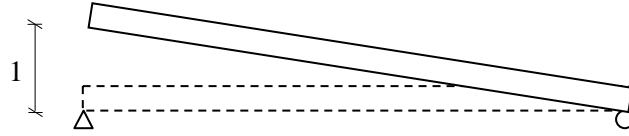
$$M_a = (30^k)(2^{ft}) + (10^k)(-0.40^{ft})$$

$$M_a = 60^{k-ft} - 4^{k-ft}$$

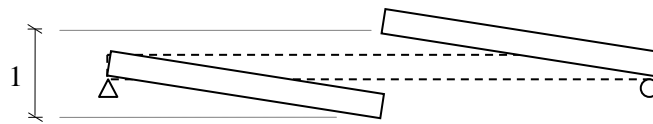
$$M_a = 56^{k-ft}$$

Influence diagrams can be constructed by applying a unit deformation associated with the type of member force.

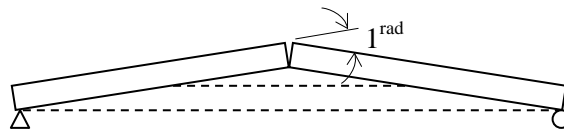
reaction



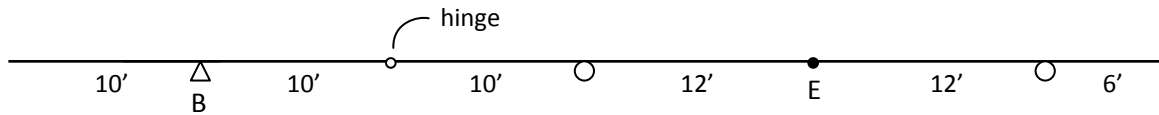
shear



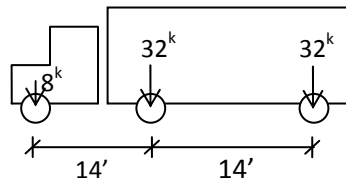
moment



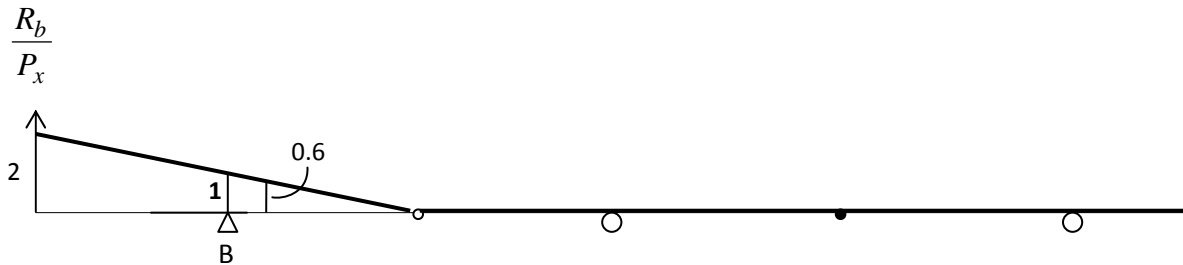
Example (Problem 8.41 in Text) A 60-foot bridge is shown below.



a) Calculate the maximum reaction at B due to the AASHTO HS20 truck, shown below.



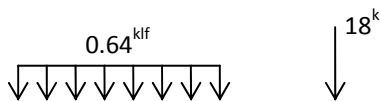
Draw the influence line for the reaction at B by displacing the beam one unit upwards at B. The deflected shape is shown above.



To calculate the reaction at B due to the truck, place the 32<sup>k</sup> rear axle at the left end of the bridge, and the 32<sup>k</sup> drive axle 14' to the right (4' to the right of B). The reaction at B is then:

$$R_B = (32^k)(2) + (32^k)(0.6) = 83.2^k$$

b) Calculate the moment at E due to the AASHTO uniform load plus concentrated load shown below.



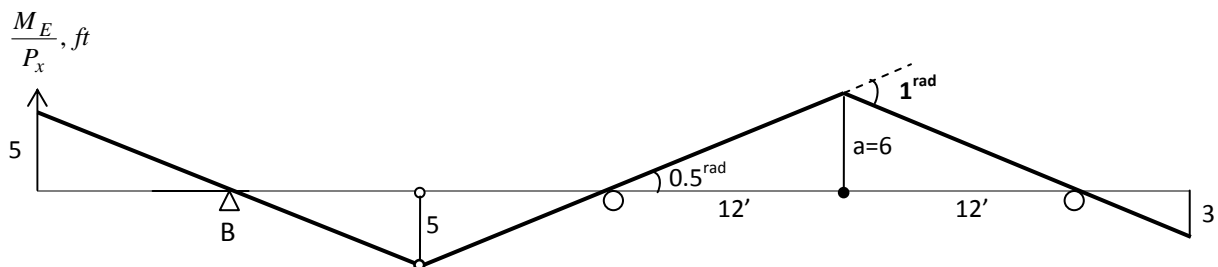
Draw the influence line for moment at E by “breaking” the beam at E and rotating the right end  $1^{\text{radian}}$  relative to the left end, as shown. Since the beam segments are both 12 feet on either end of the break, the angles of each end are equal and equal to one half of  $1.0^{\text{rad}} = 0.5^{\text{rad}}$ . The ordinate of the influence line at E is calculated from the following equation:

$$\tan \theta = \frac{a}{12'}$$

$\tan \theta = \theta$  for small displacements

$$0.5^{\text{rad}} = \frac{a}{12'}, \quad \therefore a = 6'$$

The other ordinates are calculated using similar triangles.



The moment at E due to the concentrated load is maximum when the load is placed at E. The moment due to the  $18^{\text{k}}$  load is then calculated from:

$$M_E^{18^{\text{k}}} = (18^{\text{k}})(6^{\text{ft}}) = 108^{\text{k-ft}}$$

The moment at E due to the uniform load is maximum when the load is placed at on the beam where the influence diagram is positive. The moment due to the  $0.64^{\text{k/ft}}$  load is then calculated from:

$$M_E^{0.64^{\text{k/ft}}} = (0.64^{\text{k/ft}}) \left( \frac{1}{2} (10') (5^{\text{ft}}) \right) + (0.64^{\text{k/ft}}) \left( \frac{1}{2} (24^{\text{ft}}) (6^{\text{ft}}) \right) = 62.1^{\text{k-ft}}$$

$$\therefore M_E = 108^{\text{k-ft}} + 62.1^{\text{k-ft}} = 170.1^{\text{k-ft}}$$