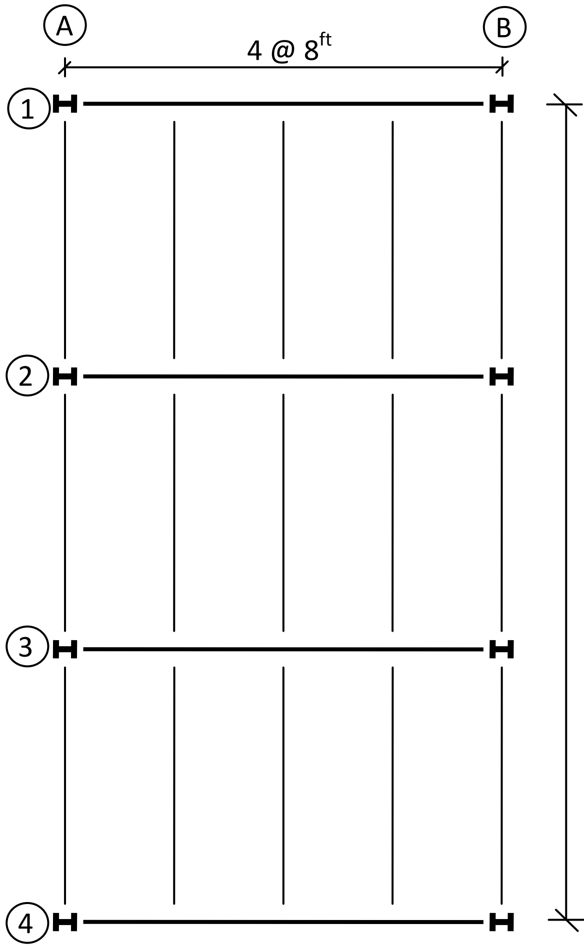
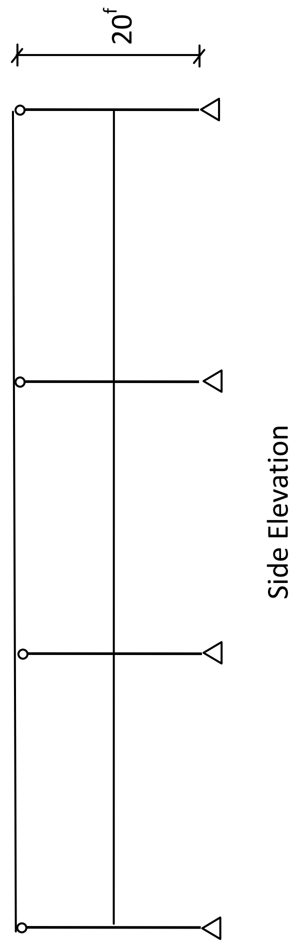
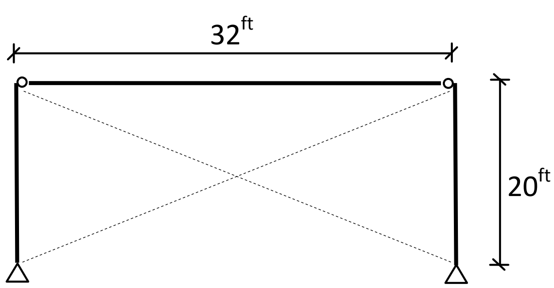
Check the strength of each type of member in the one-story steel-frame building below.



Plan View





Front Elevation

$F_y = 50^{ksi}$ all members
$F_u = 65^{ksi}$

	Shape
Purlins	W12x40
Girders	W21x44
Columns	W16x36

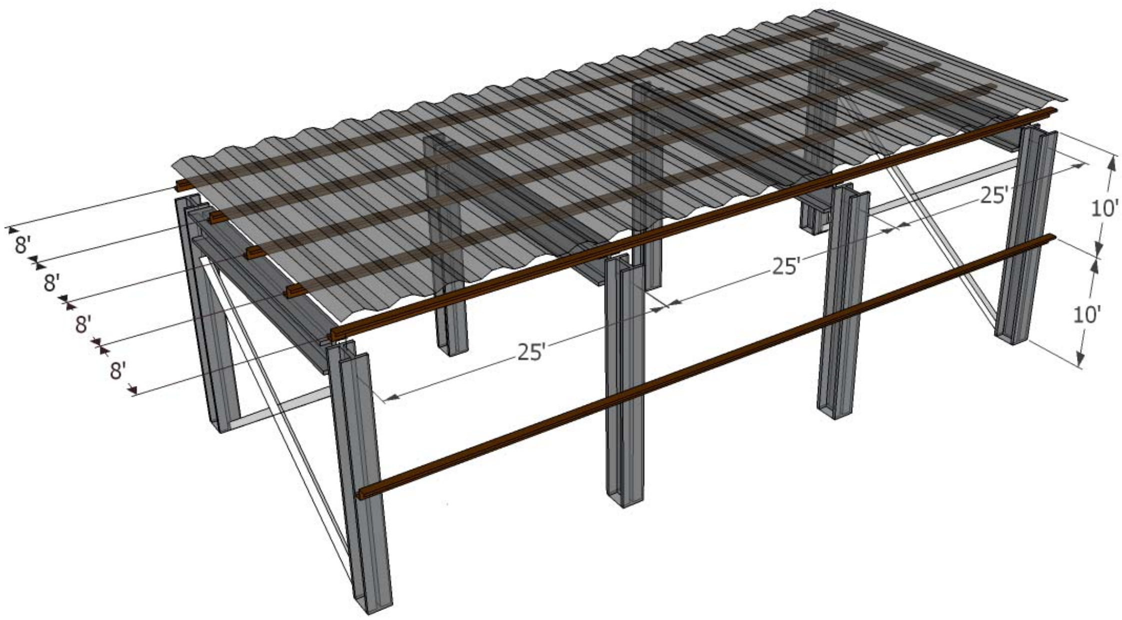
Loads:

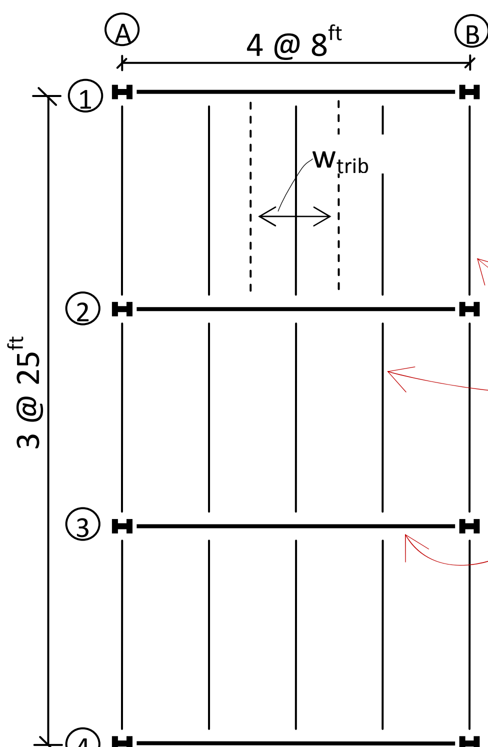
- 3.5” thick light-weight concrete slab (unit weight = 120 pcf)
- LL = 40 psf
- WL = 30 psf

Load Combinations:

- 1.2D + 1.6L
- 1.2D + 1.6W

Identify purlins (or joists) and girders. The roof deck shown below is supported by the Z-shaped purlins, which run transverse to the deck corrugations (see “exploded” building in figure below). For the example on Page 1, the owner wants to add another story at a later date, so the “roof” is a 3.5 inch thick concrete floor slab. Floors are supported by joists (in this case W12x40 steel wide flange beams). The joists are in turn supported by the girders, which run transverse to the joists. The girders are supported at their ends by the columns.





Plan View

First of all, the “H”-shaped symbols represent the columns.

In the Plan View at left, we see that some of the “vertical” members in the sketch are attached directly to columns,

but that some of the “vertical” members are attached at their ends to other beams.

The ends of the “horizontal” members, on the other hand, all attach to columns.

Therefore, the horizontal members in the sketch are girders, and the vertical members are joists.

Joist –max M_u

The joists are three-span continuous beams. Loading all of the spans of a continuous beam may not cause the maximum bending moment. Although the position of the dead load is given, the position of live load is variable and the structural engineer must determine the loading causing the maximum bending moment.

One way of determining the loading causing the maximum bending moment is to apply all possible load configurations, one at a time, and select the loading causing the maximum effect. In this class we will take a short cut that provides the same answer most of the time: we will assume that the location of the maximum bending moment due to dead plus live loads is the location with the maximum bending moment due to dead loads.

Location of max M^{D+L} = Location of max M^D

The statement above is true for continuous beams with equal span lengths.

Our procedure for calculating the maximum moment due to factored loads will be:

1. Apply the dead load to all spans and calculate the moment (M^D) using charts from the AISC manual

2a. Assume that the location of the max M^{D+L} = the location of the max M^D. Draw the influence diagram for moment for this location.

2b. Apply the live load to the spans indicated by the influence diagram and calculate the moment (M^L) using the AISC charts.

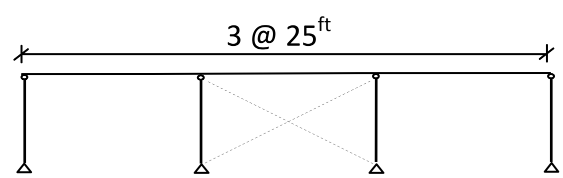
3. Calculate M_u from 1.2 M^D + 1.6 M^L.

<u>1. Dead Loads:</u>

weight of slab = 3.5"/12"/' x 120pcf = 35 psf

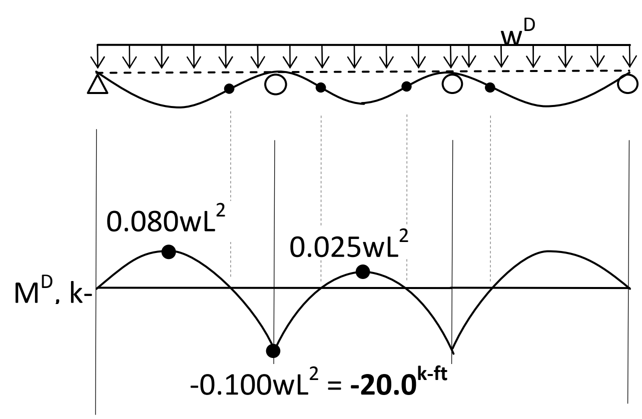
self-weight of W12x40 joist = 40 plf

↖wt in plf



		Trib. Width*	Load on Joist
slab	35 psf	8 ft	= (35 psf)(8 ft) = 0.280 klf
Joists	40 plf		= 0.040 klf
			$w^D = \Sigma = 0.320$ klf

*see sketch on bottom of Pg. 2



Max $M^D = 0.100\ wL^2$ from AISC charts

$M^D = 0.100\ (0.320^{klf})(25^{ft})^2$

<u>**M^D= 20.0$^{k\text{-}ft}$**</u>

2. Live Loads:

Assume A_T = area supported by one span of the joist (conservative)

$$LL_{reduc\_factor} = \left(0.25 + \frac{15}{\sqrt{k_{LL}\,A_T}}\right), \quad 0.4 \le LL_{reduction} \le 1.0 \qquad \text{(pg 143 FE Reference)}$$

$$k_{LL} = 2\ (beams)$$

$$A_T = \text{tributary area of joist} = (8^{ft})(25^{ft}) = 200^{sf}$$

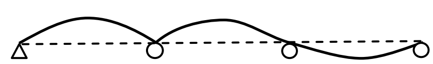
$$LL_{reduc\_factor} = \left(0.25 + \frac{15}{\sqrt{(2)(200^{sf})}}\right) = 1.00$$

Therefore

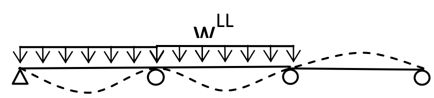
$w^L = (LL_{reduc\_factor})(LL)(\text{tributary width}) = (1.0)(40.0^{psf})(8^{ft})/(1000^{lb/k}) = 0.320^{klf}$

2a. Assume max M^{D+L} occurs at location of max M^D

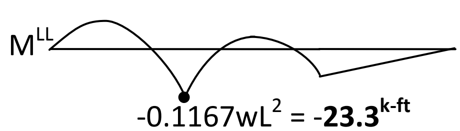
Influence Diagram for M at Support 2:



2b. Span loading to cause max. -M at Support



max $M^L = -0.1167(0.320^{klf})(25^{ft})^2 = -23.3^{k\text{-}ft}$



$M^L = 23.3^{k\text{-}ft}$

3. M_u = moment due to factored loads

Use Load Combination for gravity loads (dead and live loads) from page 1: 1.2 D + 1.6 L

$M_u = 1.2(-20.0^{k\text{-}ft}) + 1.6(-23.3^{k\text{-}ft})$ **$M_u = 61.3^{k\text{-}ft}$**

Joist –unity check

The unity check is the ratio of the demand (M_u in this case) over capacity (ϕM_n)

ϕ is the strength reduction factor for flexure, and M_n is the nominal flexure strength. ϕM_n is called the available flexure strength.

We will consider two failure modes for steel beams:

- material failure (yielding) and
- buckling (lateral-torsional buckling or LTB) in which the compression flange buckles laterally and causes the beam to twist.

The controlling failure mode depends on the lateral unbraced length of the beam's compression flange, L_b. Large unbraced lengths lead to stability failure (LTB). If the unbraced length is short enough to prevent LTB, then the beams cross-section will yield completely forming a plastic hinge in the beam. The available plastic moment strength is denoted ϕM_p.

The equations from the FE Reference for calculating ϕM_n are shown at right (pg 150):

$$L_b \le L_p: \phi M_n = \phi M_p$$

$$L_p < L_b \le L_r:$$

$$\phi M_n = C_b\left[\phi M_p - \left(\phi M_p - \phi M_r\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right]$$
$$= C_b\left[\phi M_p - BF\left(L_b - L_p\right)\right] \le \phi M_p$$

Since the joist compression flange is braced laterally continuously by the roof diaphragm, $L_b = 0$.

So: $L_b = 0 < L_p$ and $\phi M_n = \phi M_p$

$\phi M_p = 214^{k\text{-}ft}$, [AISC Table 3-2, pg 154 FE Ref.]

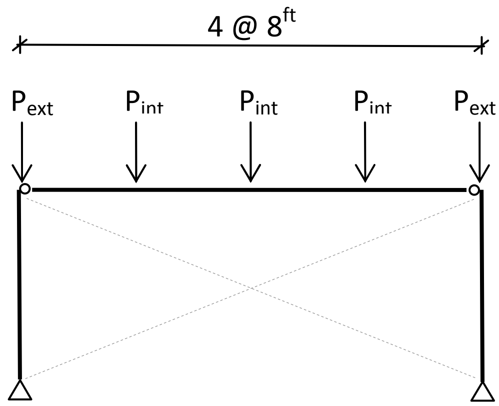
$$U.C. = \frac{M_u}{\phi M_n} = \frac{61.3^{k-ft}}{214^{k-ft}} = 0.29 < 1.0,\ OK$$

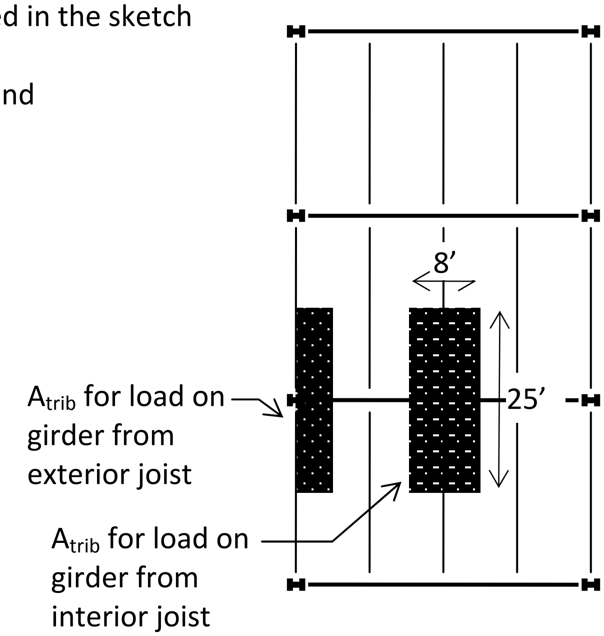
Girder –max M_u

Since all of the girders are the same size, the girder with the largest unity check will be the girder with the largest loads. Therefore, analyze a girder from an interior frame. The loads on this girder are indicated in the sketch below, where

P_{ext} = the loads on the girder from exterior joists and

P_{int} = the loads on the girder from interior joists





The tributary areas used to calculate these loads are shown in the sketch at right. Since only the interior loads will cause bending of the girder for this example, we only need to calculate P_{int}.

Dead Loads:

weight of slab = 35 psf

self-weight of joist = 40 plf

self-weight of W21x44 girder = 44 plf

(↖ wt in plf)

		Trib. Area or Trib. Length	Load on Joist
slab	35 psf	= (8 ft)(25 ft) = 200 sf	= (35 psf)(200 sf) = 7.00 k
Joists	40 plf	25 ft	= (40 plf)(25 ft) = 1.00 k
Girder	44 plf	8 ft	= (44 plf)(8ft) = 0.352 k

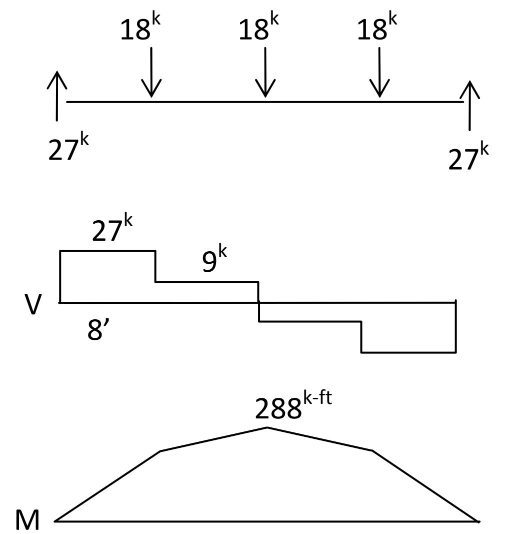
$P^D = \Sigma = 8.35$ k

Live Loads:
Since the girder is a single span, there is no need to consider span load patterns for live load.

For live-load reduction, tributary area (A_t) equals tributary width (25') ***times the span length*** (32').

$A_t = 25^{ft} \times 32^{ft} = 800^{sf}$

$LL_{reduction} = 0.25 + \frac{15}{\sqrt{(2)(800^{sf})}} = 0.625$ (between 0.4 and 1.0, OK)



$P^L = (40^{psf})(0.625)(25^{ft})(8^{ft}) = 5.0^k$

Mu:

$P_u = 1.2(8.35^k) + 1.6(5.0^k) = 18.0^k$

After drawing the shear and moment diagrams (at right),

$M_u = 288^{k\text{-}ft}$

Girder –unity check

Use the same equations on pg 150 of the FE reference as were used for the joist.

The top flange of the simply-connected girder is in compression. The joists are connected to the top flange of the girder and provide lateral restraint. Therefore the unbraced length of the compression flange, L_b, equals 8 ft.

$L_p = 4.45^{ft}$, $L_r = 13.0^{ft}$ [AISC Table 3-2, pg 154 FE Ref.]

$L_p < (L_b = 8^{ft}) < L_r$

$\therefore \ \phi M_n = C_b [\phi M_p - BF(L_b - L_p)] < \phi M_p$

$C_b = 1.0$ (always for this class)

$\phi M_p = 358^{k\text{-}ft}$ [AISC Table 3-2, pg 154 FE Ref.]

$BF = 16.8^k$ [AISC Table 3-2, pg 154 FE Ref.]

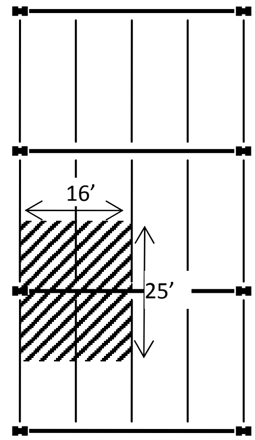
$\phi M_n = (1) [358^{k\text{-}ft} - 16.8^k(8^{ft} - 4.45^{ft}]$ [pg 150, FE Ref.]

$\phi M_n = 298^{k\text{-}ft}$ ($< 358^{k\text{-}ft} = \phi M_p$)

$UC = \frac{M_u}{\phi M_n} = \frac{288^{k-ft}}{298^{k-ft}} = 0.97 < 1$, OK

Column –max P_u (critical column = interior column)

Dead Loads:
weight of slab = 35^{psf}
$wt_{joists} = 40^{plf}$
$wt_{girder} = 44^{plf}$



		Trib. Area or Trib. Length	n	Load on Joist
slab	35 psf	= (16 ft)(25 ft) = 400 sf		= (35 psf)(400 sf) = 14.00 k
Joists	40 plf	25 ft	2.5	= (40 plf)(25 ft)(2.5 joists) = 2.50 k
Girder	44 plf	16 ft		= (44 plf)(16 ft) = 0.704 k
				$P^D = \Sigma = 17.20$ k

Live Loads:

$A_t = (32'/2)\,(25') = 400^{sf}$

$$LL_{reduction} = \left(0.25 + \frac{15}{\sqrt{(4)(400^{sf})}}\right) = 0.625$$

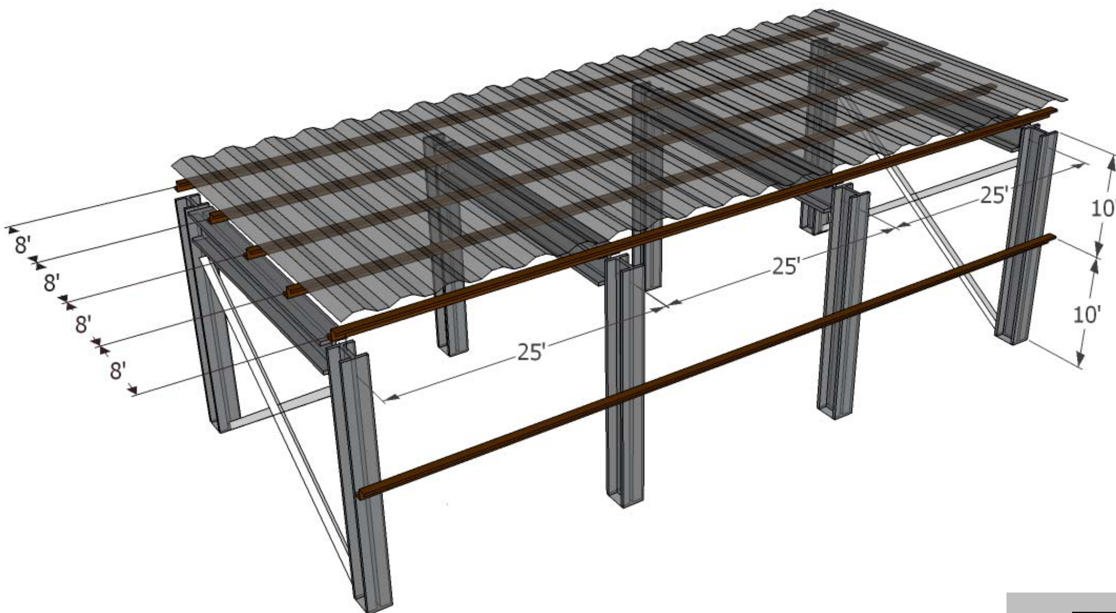
$P^L = (40^{psf})(0.625)(400^{sf}) = 10.0^k$

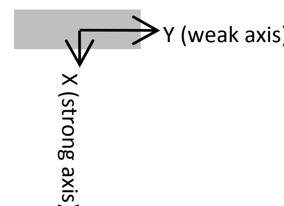
$P_u = 1.2(17.2^k) + 1.6(10.0^k)$ **$P_u = 36.6^k$**

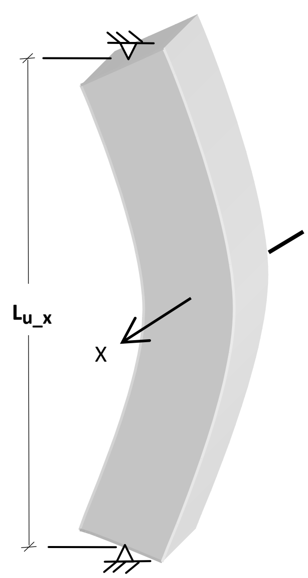
Column –unity check

Unbraced Lengths:

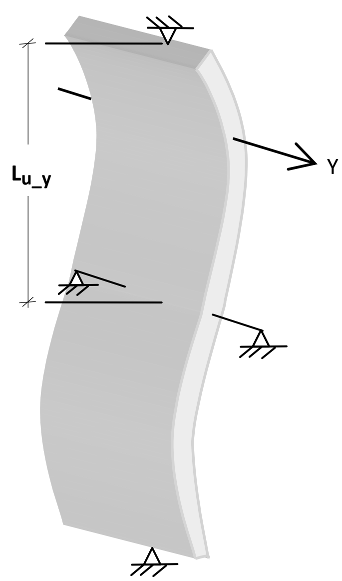
The axial strength of the column depends on its unbraced lengths. The column is braced on at its ends for buckling about its X (strong) axis (see figures below). The girt braces the column at mid-height for buckling about its Y (weak) axis. Therefore, L_{u_x} = 20 ft, and L_{u_y} = 10 ft.







Buckling about X (strong) axis



Buckling about Y (weak) axis

The relevant section properties for the column can be looked up in a table in the FE Reference.

$A = 10.6\,in^2,\ r_x = 6.51^{in},\ r_y = 1.52^{in},$ [Table1 – 1, pg 153 FE Refer]

The column buckling strength is a function of the slenderness ratio (K L / r); the higher the slenderness ratio, the lower the buckling strength. The slenderness ratio must be calculated for buckling about each axis, with the largest slenderness ratio controlling.

$$\frac{k_x\,L_{u\_x}}{r_x} = \frac{(1.0)(20^{ft}\,\frac{12^{in}}{1^{ft}})}{6.51^{in}} = 36.9$$

$$\frac{k_y\,L_{u\_y}}{r_y} = \frac{(1.0)(10^{ft}\,\frac{12^{in}}{1^{ft}})}{1.52^{in}} = 78.9$$ ← controls

KL/r = 79 (round up)

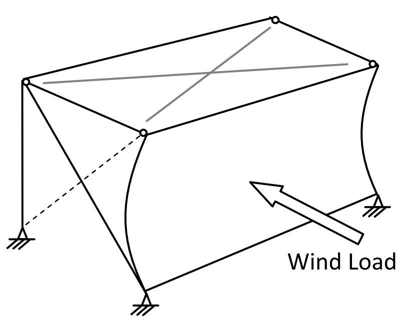
$\phi\ F_{cr} = 28.5^{ksi}$ [AISC Table 4-22, pg 157 FE Ref.]

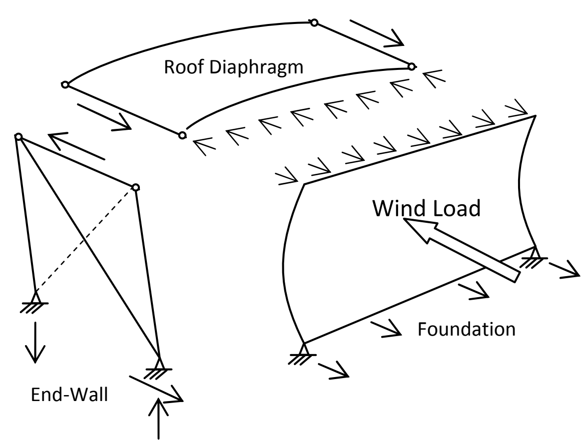
$\phi\ P_n = \phi\ F_{cr}\ A = (28.5^{ksi})\ (10.6in^2) = 302^k$

$$UC = \frac{P_u}{\phi P_n} = \frac{36.6^k}{302^k} = 0.12$$ < 1.0, OK (but over-designed)

End Wall Cross-Bracing –max T_u due to Wind Loads

Lets follow the wind loads applied to a long wall of the building (see sketch below). Assume that the wall acts like a simply-supported beam spanning between the foundations on the bottom and the roof diaphragm on the top, then half of the wind load goes to the foundations along the bottom of the wall, and the other half is distributed to the roof diaphragm.

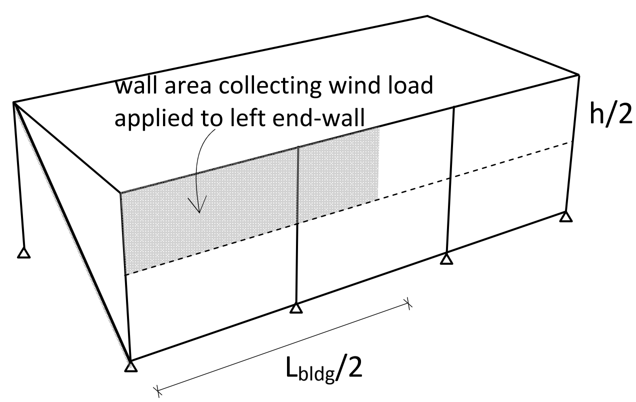




The roof diaphragm acts like a beam in the horizontal plane: it supports the distributed load from the top of the wall, and is in turn supported at its ends by the end walls. Assume that only the end-walls are braced against sidesway (x-bracing on the internal frames would restrict the use of the building). Then half of the wind load to the roof diaphragm is applied to the top of each end wall.

The end-wall acts like a vertical cantilever beam: the horizontal force applied to its free end results in a horizontal reaction at its base and a couple (called the "overturning moment" due to wind load). Rather than specify one diagonal brace that can carry tension or compression (depending on the wind direction), structural engineers usually specify x-bracing and assume that the diagonal brace in compression buckles elastically. Therefore the horizontal wind load at the top of the end-wall is resisted by the horizontal component of the tension diagonal brace.

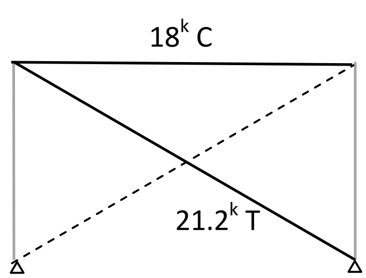
The portion of the long wall area that collects the wind load distributed to the top of an end-wall is indicated in the sketch below.



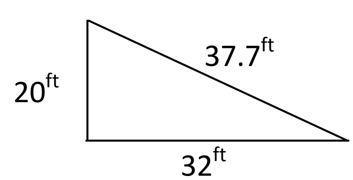
Isometric View

For the current example, the force at the top of the end-wall due to factored wind load (see load combination on Page 1) is:

$$P^{W}_{U\,end\ wall} = (1.6)(WL)\left(\frac{h_{col}}{2}\right)\left(\frac{Length\ of\ \ Bldg}{2}\right) = (1.6)(30\,psf)\left(\frac{20'}{2}\right)\left(\frac{75'}{2}\right) = 18.0^{k}$$



End-Wall Elevation



The tension force in the diagonal brace due to factored wind loads is therefore:

$T_{U\ end\text{-}wall\ brace} = (18^{k})(37.7^{ft} / 32^{ft})$ **$T_{U\ end\text{-}wall\ brace} = 21.2^{k}$**