These notes present the procedures for analyzing beams. Beams are members loaded transversely subject to flexure and shear. Chapters F and G of the Specifications, pg 16.1-44 – 69. Members subject to combined flexure and compression will be covered later (Chapter H in the Specifications). These notes will focus on I-shaped members.

**Failure Modes**
As with compression members, members in flexure can fail in three different ways:

1. **Material Failure (Yielding)** leading to a plastic hinge,
2. **Stability Failure** over the length of the member called lateral-torsional buckling, and
3. **Stability Failure** of a plate-shaped cross-section element of the member (local buckling).

1. **Plastic Hinge.** When normal stresses due to flexure exceed the yield stress at all points in the beam cross-section, a plastic hinge is said to form. Although the load and associated bending moment required to just form the plastic hinge can still be supported, additional bending moment will cause unlimited deformation at the hinge, leading to redistribution of load for indeterminate structures, and collapse for determinate structures such as the simply-supported beam in Figure 1.

![Diagram of a plastic hinge](image)

**Figure 1.** Formation of a plastic hinge

2. **Lateral-torsional Buckling.** Flexure causes compressive and tensile normal stresses. For example, gravity loads on a simply-supported beam will cause compression in the top of the beam, and tension in the bottom. The compression in the top of the beam can cause the top of the beam to buckle laterally. Since the bottom of the beam is in tension, it does not buckle, causing the beam to twist about its longitudinal axis as the top buckles laterally (see Figure 2). Lateral-torsional buckling can be prevented by bracing the beam so that it cannot deflect laterally or twist.
3. Local Buckling. As with compression members, local buckling of the plate-shaped elements of the cross-section may occur. Depending on the slenderness of the plate-shaped element (width to thickness ratio, \( b/t \)), the cross section of a flexure member is classified as either compact, non-compact, or slender.

- Compact sections can develop the full plastic moment without buckling. Most W-shape sections are compact.
- Plate-shaped elements of non-compact sections will buckle inelastically before the plastic moment is reached.
- Plate-shaped elements of slender sections will buckle elastically before the plastic moment is reached.

Design Equations

1. Plastic Moment Capacity, \( M_p \). Normal stresses due to flexure are distributed linearly over the depth the cross section, as illustrated in Figure 1a below. Increased flexure causes the outer “fibers” (furthest from the neutral axis) to yield, as indicated in Figure 1b. Ultimately, all parts of the cross-section yield (Figure 1c).
Figure 1. Elastic to plastic strain and stress distributions.

The moment corresponding to Figure 1c is called the “plastic moment”, $M_p$. It can be calculated by summing forces about the neutral axis (see Figure 2).

$$M_p = \int \sigma_y \cdot y \, dA = \left[ \sigma_y \cdot \frac{A}{2} \cdot \bar{y} \right] + \left[ \sigma_y \cdot \frac{A}{2} \cdot \bar{y} \right]$$

$$M_p = \sigma_y \cdot A \cdot \bar{y}$$

where $A$ is the gross area of the section and $\bar{y}$ is the distance from the neutral axis to the centroid of the half-section.
Using AISC terminology, the plastic moment, $M_p$ is

$$M_p = F_y \cdot Z_x$$

Eqn. F2-1

Where $Z_x$ is called the plastic section modulus.

2. Lateral Torsional Buckling (LTB). Lateral torsional buckling is a function of the unbraced length, $L_b$. The unbraced length is the distance between lateral supports to the compression flange. In Figure 3 below, the center beam has three unbraced segments with $L_b = 10'$, 15' and 10', respectively.

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**Figure 2.** Compressive and tensile stresses on section corresponding to plastic moment.

**Figure 3.** Unbraced lengths ($L_b$) for lateral torsional buckling.
The AISC equations for the nominal flexure strength, $M_n$, for a beam subject to lateral torsional buckling are summarized in Figure 4 below. $M_n$ depends on the unbraced length, $L_b$. If $L_b$ is less than $L_p$ ($p$ for plastic), then the full plastic moment ($M_p$) can be developed. If $L_b$ is greater than $L_\text{r}$, then the beam will fail in elastic lateral torsional buckling. If $L_b$ is between $L_p$ and $L_\text{r}$, then the beam will fail in inelastic lateral torsional buckling.

\[
\phi M_n = \phi M_p \\
[ \phi M_p - BF (L_b - L_p) ] \leq \phi M_p
\]

\[0.7 F_y S_x\]

**Figure 4.** Design equations for Lateral Torsional Buckling (LTB)

The equations for $M_n$ with LTB were derived assuming a constant magnitude bending moment distribution along the beam. $M_n$ for unbraced beam segments with non-uniform bending moment diagrams will be higher. This is accounted for with the $C_b$ factor, which is similar to the effective length factor ($k$) for axial buckling (see Sect. F1, pg 46). Values for $C_b$ for common lateral brace configurations are given in Table 3-1 (pg 3-10) in the Manual. For beams with uniform loads, $C_b$ can usually be conservatively yet reasonably assumed to be equal to 1.0.

3. **Compression Flange Local Buckling (FLB).** Local buckling of the compression flange (FLB) occurs if the flange is so slender that it buckles before the plastic moment can be developed. The AISC equations for $M_n$ with FLB are summarized in Figure 5 below. $M_n$
depends on the slenderness of the flange \( \lambda = \frac{b_f}{2t_f} \). If \( \lambda \) is less than \( \lambda_{pf} \), then the full plastic moment can be developed and the section is considered to be compact. If \( \lambda \) is greater than \( \lambda_{rf} \), then the flange will buckle elastically and the section is considered to be slender. If \( \lambda \) is between \( \lambda_{pf} \) and \( \lambda_{rf} \), then the flange will buckle inelastically and the section is considered to be noncompact.

**Figure 5.** Design equations for \( M_n \) with compression flange local buckling.