The first step in analyzing a truss is to determine if the truss is stable or unstable. The following pages contain:

1. A Demonstration of Stability and Determinancy
2. Advantages and Disadvantages of Determinate and Indeterminate Trusses
3. Procedures for Determining Stability and Determinancy

1. Demonstration of Unstable, Stable & Determinate, and Indeterminate

In this section, a simple truss is used to demonstrate stability and determinancy.

The truss in Figure 1a below is not stable, and is therefore not a structure. The joints of an ***unstable*** structure can be displaced without causing any of the members to deform (lengthen or shorten in this case), as shown in Figure 1b.

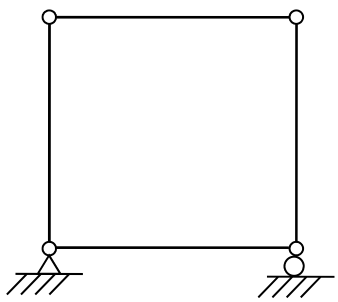


Figure 1a. Unstable Truss

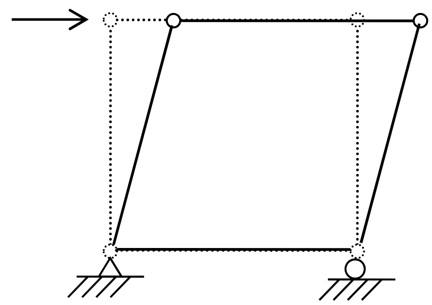


Figure 1b.

In Figure 2a, a diagonal member has been added to the truss of Figure 1. This truss is **stable**: the horizontal load shown in Figure 2b causes the diagonal to stretch and the members shown in blue to shorten. The joints cannot be displaced without causing members to deform and therefore to resist the attempted displacement. The bar forces in the truss of Figure 2b can easily be calculated using the equations of static equilibrium and the truss is said to be ***statically determinate***.

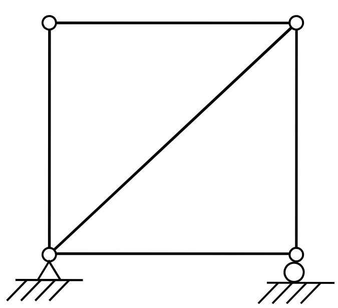


Figure 2a. Stable & Determinate Truss

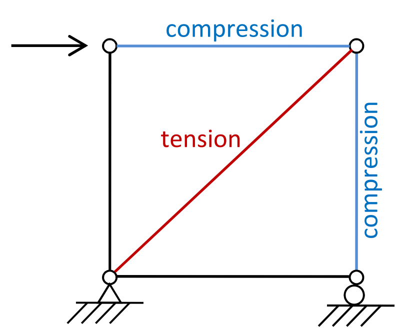


Figure 2b.

In Figure 3 below we have added another member to the truss of Figure 2. The bar forces of this truss cannot be calculated using the equilibrium equations alone and the truss is said to be ***statically indeterminate***. The forces can be calculated by hand for the truss in Figure 3, but it is an involved procedure. Most practicing structural engineers would calculate the bar forces using a structural analysis computer program.

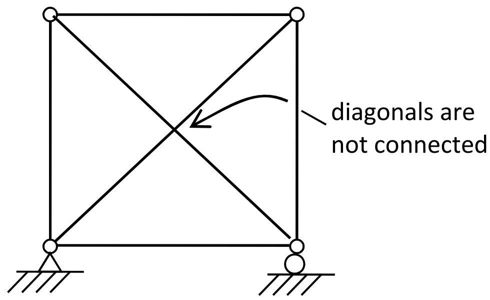


Figure 3a. Stable & Indeterminate Truss

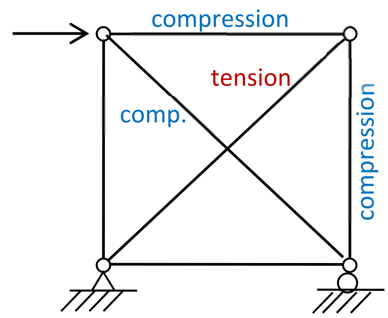


Figure 3b.

2. Advantages and Disadvantages of Indeterminate Trusses.
Statically indeterminate trusses differ from statically determinate trusses in several ways besides complexity of analysis by hand calculations.

- **Indeterminate trusses have redundant load paths.** As illustrated by the trusses in Figures 1 through 3, the indeterminate truss of Figure 3 has one additional member than is required for stability. If one of the diagonals in Figure 3 fails, for example, the load will be shifted to the other diagonal and the truss will not collapse. If the single diagonal of Figure 2 fails, however, the truss will collapse.
- **Indeterminate trusses can be more structurally efficient** (where structural efficiency = load at failure / weight of structure). Two similar trusses are shown below. In Figure 4a, two simple-span trusses are shown; and if Figure 4b, a two-span continuous truss is shown.

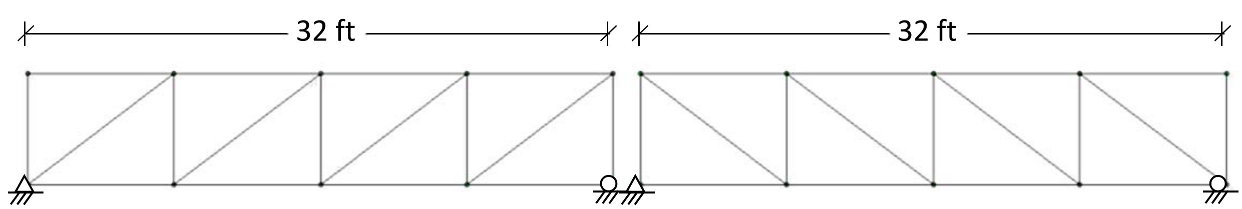


Figure 4a. Two simple-span trusses

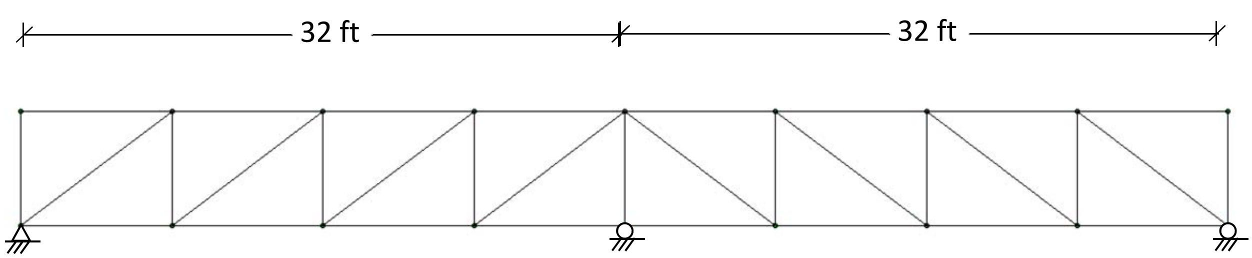


Figure 4b. A two-span continuous truss

Identical loads are applied to the top of both trusses. As seen in Figure 5 below, the maximum compressive bar forces in the continuous trusses are only 67% of the maximum compressive bar forces in the simple-span trusses.

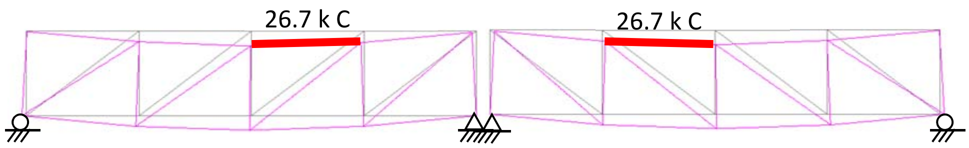


Figure 5a. Maximum compressive bar forces in the two simple span truss

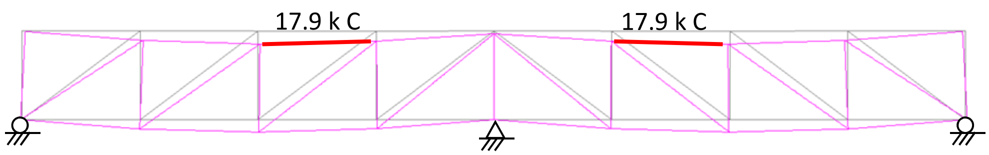
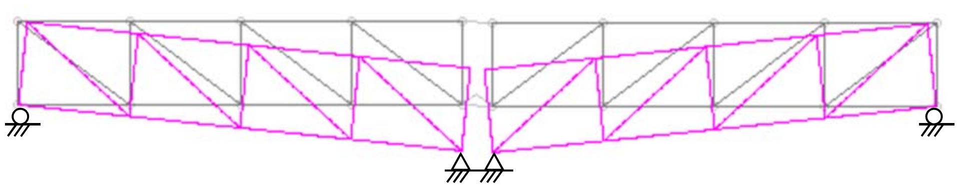


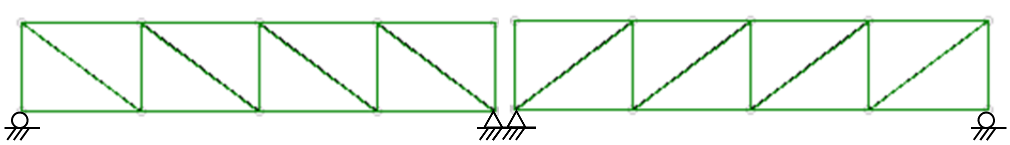
Figure 5b. Maximum compressive bar forces in the two-span continuous truss

- **Indeterminate trusses are much more affected by settlement.**

For statistically determinate structures, support settlement does not cause member deformation (see Figure 6a below); and therefore support settlement does not induce member forces (see Figure 6b below).



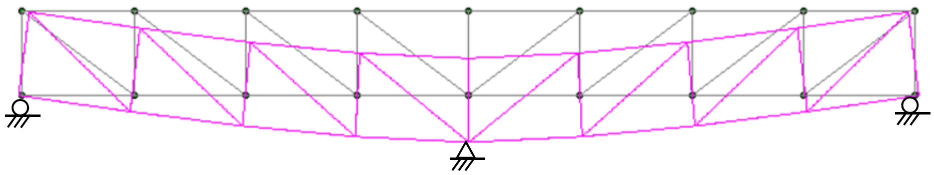
(a)



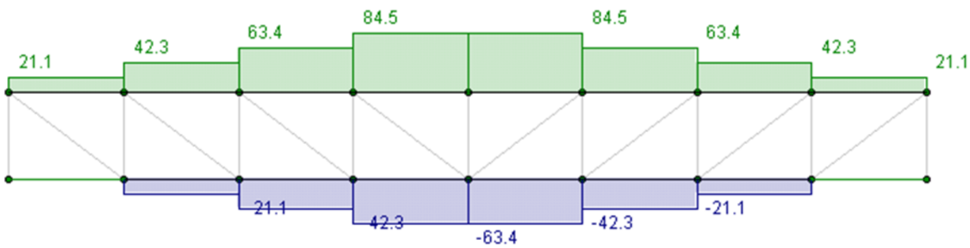
(b)

Figure 6. Deflected shape (a) and chord forces (b) in kips of two simple-span trusses due to 1" settlement of center support.

For statistically indeterminate structures, support settlement *does cause* member deformation (see Figure 7a below); and therefore support settlement *does induce* member forces (see Figure 7b below).



(a)



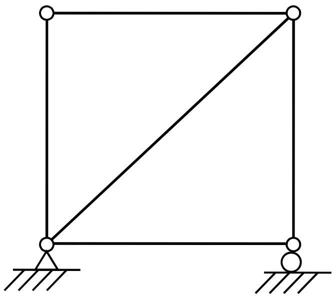
(b)

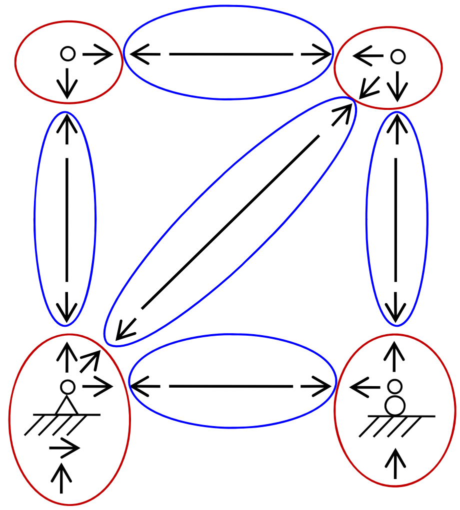
Figure 7. Deflected shape (a) and chord forces (b) in kips of two-span continuous truss due to 1” settlement of center support.

3. Procedures for Determining Stability and Determinacy.

Procedures are presented below for determining if a structure is stable or unstable, and determinate or indeterminate. The first procedure involves counting the number of unknown forces, counting the number of equilibrium equations, and comparing the two numbers. This procedure is fairly straightforward to follow but is not foolproof, since it does not account for the arrangement of members and supports. The second procedure is harder to implement, but serves as a good check on the results from the first procedure.

Counting Procedure. The procedure is derived using the simple truss below left. The truss is "exploded" into nine separate free-body diagrams, one for each member and one for each joint.





Number of Unknowns = 13 forces

- 2 forces per member x 5 members = 10
  (the forces are the joints are equal and opposite to the adjacent member-end force and are not counted)
- 3 reactions

Number of Equations = 13

- 1 equation per member x 5 members = 5 ($\Sigma F_{||}$)
- 2 equations per joint x 4 joints = 8 ($\Sigma F_{H,}$ ΣF_V)

Shortcut: (Used by some books)
Number of Unknowns = ~~13~~ **8**

- ~~2~~ **1** forces per member x 5 members = ~~10~~ **5**
- 3 reactions

Number of Equations = ~~13~~ **8**

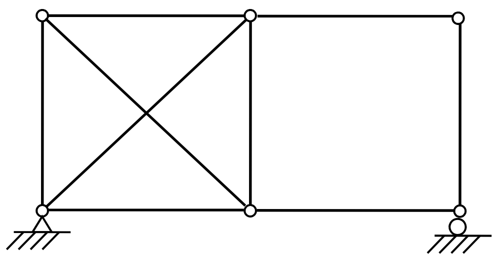
- ~~1 equation per member x 5 members = 5~~
- 2 equations per joint x 4 joints = 8

Since the number of unknown forces = the number of equations, the structure is statically determinate (the member forces can be calculated using equilibrium equations).

In general:

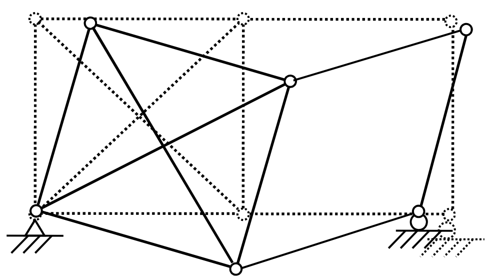
	If		**The structure is**
number of unknowns	<	number of equations	Unstable
number of unknowns	=	number of equations	Stable & Determinate
number of unknowns	>	number of equations	Indeterminate

The procedure outlined above does not always work with regard to stability. For example, the truss below has the same number of unknowns as equations, yet it is unstable as illustrated in the second figure below.



Number of Unknowns
= 9 member forces + 3 reactions = 12

Number of Equations
= 6 joints x 2 eqns/joint = 12



The structure displaces without any members deforming.

No member deformation means no resistance to structure displacement.

The structure is therefore unstable.

Displaced Shape Procedure. The figure above suggests an alternate procedure of determining whether a structure is stable or unstable.

- If a displaced shape of the structure can be drawn so that no members deform, the structure is unstable.
- If a displaced shape cannot be drawn without causing a member to deform, the structure is stable.

The method can be extended to also determine if a structure is statically indeterminate.

- If removal of one constraint (a support or a member force) causes the structure to be unstable (i.e. the structure can be displaced without deforming a member), then the original structure is stable and determinate (see Figure 2 for an example).
- If two or more constraints need to be removed to cause the structure to be unstable, the original structure is stable and indeterminate (see Figure 3 for an example).