A structure is statically indeterminate if the member forces cannot be calculated using the equations of static equilibrium.

Example 1.

Number of Unknowns = 16
- 6 forces per member \(\times\) 2 members = 12
- 4 reactions

Number of Equations = 16
- 3 equations per member \(\times\) 2 members = 6 \((\Sigma F_H, \Sigma F_V, \Sigma M)\)
- 3 equations per joint \(\times\) 3 joints = 9 \((\Sigma F_H, \Sigma F_V, \Sigma M)\)
- 1 equation of condition at moment release \((\Sigma M)\)

Shortcut:

Number of Unknowns = 16 10
- 3 forces per member \(\times\) 2 members = 12 6
- 4 reactions

Number of Equations = 16 10
- 3 equations per member \(\times\) 2 members = 6 \((\Sigma F_{HR}, \Sigma F_{VT}, \Sigma M)\)
- 3 equations per joint \(\times\) 3 joints = 9 \((\Sigma F_H, \Sigma F_V, \Sigma M)\)
- 1 equation of condition at moment release \((\Sigma M)\)

Since the number of unknowns = the number of equations, the structure is statically determinate (member forces can be calculated using equilibrium equations).
In general:

<table>
<thead>
<tr>
<th>If</th>
<th>The structure is</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of unknowns</td>
<td>number of equations</td>
</tr>
<tr>
<td>&lt;</td>
<td>Unstable</td>
</tr>
<tr>
<td>=</td>
<td>Stable &amp; Determinate</td>
</tr>
<tr>
<td>&gt;</td>
<td>Indeterminate</td>
</tr>
</tbody>
</table>

The procedure outlined above does not always work with regard to stability. An alternate method (the displaced shape method) of determining whether a structure is stable or unstable, and determinate or indeterminate is the following:

- If the displaced shape of the structure can be drawn so that no members deform, the structure is unstable.
- If the displaced shape cannot be drawn without causing a member to deform, the structure is stable.
  - If removal of one constraint (a support force or a member force) causes the structure to be unstable (i.e. the structure can be displaced without deforming a member), then the original structure is stable and determinate.
  - If two or more constraints need to be removed to cause the structure to be unstable, the original structure is stable and indeterminate.

Example 2.

Counting Method

Number of Unknown Forces = (2 members) x (3 member forces / member) + 4 reactions = 10
Number of Equilibrium Equations = (3 joints) x (3 equations / joint) = 9
Therefore structure is indeterminate. It has one more force (called a redundant force) than needed for a determinate structure.
**Displaced Shape Method**

1. Original Structure:

Structure displacement causes member deformation, therefore this structure is stable.

2. Original Structure minus 1 force

Structure displacement causes member deformation, therefore this structure is stable.

3. Original Structure minus 2 forces

Structure displacement causes NO member deformation, therefore this structure is UNstable.

“Case 2. Original Structure minus 1 force” is therefore the stable and determinate version of this structure. *Therefore the original structure is indeterminate with one redundant.*

Example 3.

**Counting Method**
Number of Unknown Forces = (3 members) x (3 member forces / member) + 6 reactions = 15
Number of Equilibrium Equations = (4 joints) x (3 equations / joint) + 2 moment releases = 14
Therefore structure is indeterminate with one redundant.

Displaced Shape Method
1. Original Structure:
   Structure displacement causes member deformation, therefore this structure is stable.

2. Original Structure minus 1 force
   Structure displacement causes member deformation, therefore this structure is stable.

3. Original Structure minus 2 forces
   Structure displacement causes NO member deformation, therefore this structure is UNstable.

“Case 2. Original Structure minus 1 force” is therefore the stable and determinate version of this structure. **Therefore the original structure is indeterminate with one redundant.**
Advantages of Indeterminate Structures

Indeterminate structures are often more structurally efficient.

Example 4.

Indeterminate structures also provide redundant load paths, meaning if a member fails, the load is redistributed to other members and the structure does not collapse. Say the bending strength of the middle span is exceeded and a plastic hinge forms.

The determinate structure is now unstable and will collapse, while for the indeterminate structure, load is redistributed to the other members and the structure does not collapse.

Disadvantages of Indeterminate Structures

Disadvantages of statically indeterminate structures include

- settlement (and other applied displacements such as temperature change) can cause large internal forces in indeterminate structures

- calculation of member forces is more complicated since the equilibrium equations alone are insufficient.