Darcy's Law: \( \nu = \frac{K}{c} \)

\( \nu \) = discharge velocity through saturated soils

\( K \) = hydraulic conductivity (permeability parameter)

\( c \) = hydraulic gradient = \( \frac{\Delta h}{L} \)

\( h \) = total head = \( \zeta + \frac{\nu}{\gamma w} \)

\( \zeta \) = elevation head

\( \frac{\nu}{\gamma w} \) = pressure head

Can measure pressure head with a piezometer (open stand pipe)

\( q = \nu A \)

\( q \) = flow rate

\( A \) = cross-sectional area of soil through which flow occurs

\( L \) = distance (parallel to flow) over which \( \Delta h \) occurs
Examples:

1a. Unconfined aquifer, level

\[ \Delta h = h_a - h_b = 0, \quad \Rightarrow \quad \frac{\Delta h}{h_a} = 0, \quad \Rightarrow \quad \phi = \theta = 0 \]

1b. Unconfined aquifer, inclined
\[ \Delta h = h_a - h_b \]
\[ q = \frac{1}{m} A = \frac{1}{\Delta z} \cdot \frac{1}{L} \]
\[ \Delta h = 5 \tan \alpha \]
\[ L = \frac{5}{\cos \alpha} \]
\[ A = (H \cos \alpha) (1 + \text{out-of-page}) \]

\[ q = \frac{K}{S} \frac{\text{tan} \alpha \cdot \cos \alpha}{H \cos \alpha} \]
\[ q = \frac{K}{S} \sin \alpha \cdot H \cos \alpha \]

2a. Confined aquifer, level

\[ \Delta h = h_a - h_b, \quad q = \frac{1}{m} \frac{\Delta h}{L} \quad H \quad (1 \text{ ft out-of-page}) \]
2b. Confined aquifer - inclined

\[ q = \frac{k \Delta h}{S \cos \alpha} \quad \text{Head} \cos \alpha (1 + \tan \alpha) \]

3. a. Constant Head permeability test

- Adjust \( q_{in} \) until \( q_{in} = q_{out} \)
- Measure vol. of water \( Q \) flowing through soil over time \( t \).

\[ Q = q \, t \]
\[ Q = \frac{k \, \Delta h}{L} \, A \, t \]
\[ k = \frac{Q \, L}{\Delta h \, A \, t} \]
3b. Falling Head permeability test

Let \( h = \Delta h \)

\( h_0, t_0 = \) head & time at start of test

\( h_i, t_i = \) \( i \)th reading during test

Set flow rate through soil = "flow" in standpipe at time \( t \)

\[ q_{\text{soil}} = k \frac{h}{L} A, \quad q_{\text{standpipe}} = -\frac{dh}{dt} A \]

\[ \frac{dh}{L} A = -\frac{dh}{dt} A \]

\[ dt = -\frac{A}{k} \frac{dh}{h} \]

\[ \int_{t_0}^{t_i} dt = -\frac{A}{k} \int_{h_0}^{h_i} \frac{1}{h} dh \]
\[
[t]_{t_0} = -\frac{aL}{Ak} \left[ \ln h \right]_{h_0}^{h_i} \\
(t_i - t_0) = -\frac{aL}{Ak} \left( \ln h_i - \ln h_0 \right) \\
t_i - t_0 = -\frac{aL}{Ak} \ln \frac{h_i}{h_0}, \quad t_i - t_0 = \frac{aL}{Ak} \ln \frac{h_0}{h_i} \\
K = \frac{aL}{A (t_i - t_0)} \ln \frac{h_0}{h_i}
\]