For a truss elements at different angles, the stiffness equations for each element must be transformed to a common coordinate system, called the system coordinate system. A horizontal and a vertical degree of freedom (DOF) is assigned at each node in the structure, as illustrated below.

The element displacements $\delta$ are written in terms of the system displacements $\Delta$ and the angle $\theta$ from the system DOF 1 to the element DOF 1 (from $\Delta_1$ to $\delta_1$).

$$\delta_1 = \Delta_1 \cos \theta + \Delta_2 \sin \theta$$

The transformation from element displacements to system displacements shown above can be written in matrix form for both ends of the member as

$$\begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$$

$$\begin{pmatrix} \delta_1 \\ \delta_2 \\ \Delta_3 \\ \Delta_4 \end{pmatrix} = [T][\Delta]$$

where $[T]$ represents the coordinate transformation matrix.
The system forces \{P\} can be written in terms of the element end forces \{p\} as shown below.

\[
P_1 = p_1 \cos \theta \\
P_2 = p_1 \sin \theta \\
P_3 = p_2 \cos \theta \\
P_4 = p_2 \sin \theta
\]

In matrix form, these equations become

\[
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 \\
P_4 
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & 0 \\
\sin \theta & 0 \\
0 & \cos \theta \\
0 & \sin \theta
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2
\end{bmatrix}
\]

\[
\{P\} = [T]^T \{p\} \quad (2)
\]

where \([T]^T\) represents the transpose of \([T]\).

The element stiffness equations in element coordinates are.

\[
\{p\} = [k']\{\delta\} \quad (3)
\]

where \([k']\) represents the 2x2 element stiffness matrix in element coordinates.

Substituting Eqn. 3 into Eqn. 2

\[
\{P\} = [T]^T \{p\} = [T]^T [k'] \{\delta\}
\]

and substituting Eqn. 1 into the above

\[
\{P\} = [T]^T [k'] [T] \{\Delta\}
\]

or

\[
\{P\} = [k] \{\Delta\}
\]

where

\[
[k] = [T]^T [k'] [T] \quad (4)
\]

\([k]\) represents the 4x4 element stiffness matrix in system coordinates.