Many reinforced concrete beams are cast at the same time the slabs they support are cast. Such beams are monolithic with the slab (monolithic comes from the Greek words mono (one) and lithos (stone)). These beams form the shape of a "T" and are composed of two rectangular sections, the flange and the web. We need four quantities to describe the dimensions of a T-beam:

- \( b_w \) = width of web
- \( b_f \) = width of flange
- depth (or thickness) of slab = \( t \)
- overall depth = \( h \)

T-beams are efficient for positive bending (compression on top of beam) because a large area in the flange is available for compression and minimal concrete exists in the cracked tension zone below the neutral axis. The compressive stresses in the flange are not evenly distributed (due to a phenomenon called "shear lag"). The total compressive force in the flange is assumed to be distributed uniformly over a flange width called the "effective" flange width.
We will divide the compression zone in a T-beam into two regions: the web, which we will analyze just as we did for a rectangular beam, and the "flange", where the flange will have a width of \( b_f - b_w \).

**Important:** The depth of the stress block in the flange will be either:

- \( a \), if \( a < t \), or
- \( t \), if \( a > t \).

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**Example of T-beam Analysis.**

**span = 27 ft**

\( w_{SDL} = 1.0 \text{ klf} \)

\( w_{LL} = 2.0 \text{ klf} \)

\( f'c = 4000 \text{ psi} \)

\( f_y = 60,000 \text{ psi} \)

6 #10 bars, \( A_s = 7.62 \text{ in}^2 \)

\( t_{stab} = 4 \text{ in} \)

\( h = 19 \text{ in} \)

\( \text{beam spacing} = 30 \text{ in} \)

\( b_w = 13 \text{ in} \)

\( \text{from spreadsheet: } K = -0.000499/\text{in} \)

\( y_t = 6.02 \text{ in} \)

Calc. \( M_u \)

\[
\begin{align*}
  w_{sw} &= \text{unit wt} \left[ (b_w)h + (\text{beam spacing} - b_w)(t_{slab}) \right] \\
  w_{sw} &= (0.150 \text{kcf}) \left[ (13')(19\prime) + (30\prime - 13\prime)(4\prime) \right] = 0.328 \text{ klf} \\
  w_u &= 1.2 (w_{sw} + w_{SDL}) + 1.6 w_{LL} \\
  w_u &= 1.2 (0.328 \text{ klf} + 1.0 \text{ klf}) + 1.6 (2.0 \text{ klf}) = 4.79 \text{ klf} \\
  M_u &= w_u (\text{span})^2 / 8 = (4.79 \text{ klf}) (27 \text{ ft})^2 / 8, \\
  M_u &= 437 \text{ k-ft} \\
\end{align*}
\]

Calc. effective flange width, \( b_f \): (ACI 8.10.2)

\[
\begin{align*}
  b_f &= \text{minimum of: } \frac{\text{span}}{4} \quad \frac{16 t_{slab} + b_w}{16} \quad \frac{\text{beam spacing}}{30} \\
  29' (12\prime / 4 = 87\prime) \quad 16(4\prime) + 13\prime = 77\prime \\
  b_f &= 30\prime 
\end{align*}
\]
Calc. stress resultants
\[ a = \beta_1 y_t = 0.85 \text{ (6.02 in)} = 5.113 \text{ in} \] (therefore compression block extends below flange)

\[ C_w = 0.85 f'c a b_w = 0.85 \text{(4000psi)}(5.113\text{''})(13\text{''}) = 226 \text{ k} \]
\[ C_f = 0.85 f'c t_{slab} (b_f - b_w) = 0.85 \text{(4000psi)}(4\text{''})(30\text{''} - 13\text{''}) = 231 \text{ k} \]

\[ T = As f_y = (7.62 \text{ in}^2)(60 \text{ ksi}) = 457 \text{ k} \]

SFH:  \( C = T? \),  \( 226k + 231k = 457k \),  OK

Calc. strain in bottom layer of steel (needed to calc. \( \phi \))
\[ \varepsilon_{sbottom\_layer} = K y_{s\_bottom\_layer}, \]
\[ y_{s\_bottom\_layer} = y_t - d_{bottom\_layer} \]
\[ d_{bottom\_layer} = h - 2.5'' = 19'' - 2.5'' = 16.5'' \]
\[ y_{s\_bottom\_layer} = 6.02'' - 16.5'' = -10.48'' \]
\[ \varepsilon_{sbottom\_layer} = (-0.000499 /\text{in})(-10.48'') = 0.0052, \]\[ \text{therefore, } \phi = 0.90 \text{ (barely)} \]

Since 0.005 \( \gg \) 0.002 = \( \varepsilon_y \), all layers have yielded

Calc. \( \phi M_n \)

Sum moments about neutral axis
\[ \phi M_n = \phi \left[ C_w (y_{C\_w}) + C_f (y_{C\_f}) + T (y_{T\_}) \right] \]
\[ y_{C\_w} = y_t - a/2 = 6.02'' - 5.113'' / 2 = 3.46'' \]
\[ y_{C\_f} = y_t - t_{slab} / 2 = 6.02'' - 4'' / 2 = 4.02'' \]
\[ y_{T\_} = d - y_t = 15.5'' - 6.02'' = 9.48'' \]

\[ \phi M_n = 0.90 \left[ 226k \text{ (3.46'')} + 231k \text{ (4.02'')} + 457k \text{ (9.48'')} \right] \text{(1' / 12'')}, \]
\[ \phi M_n = 453 \text{ k-ft} \]

\[ \phi M_n = 453 \text{ k-ft} > 437 \text{ k-ft} = M_u, \text{ OK} \]