Cast-in-place reinforced concrete structures have monolithic slab to beam and beam to column connections. Monolithic comes from the Greek words mono (one) and lithos (stone). Two consequences of monolithic construction are "T-shaped" beams and reinforcing steel in the compression zone (compression reinforcement).

A typical reinforced concrete floor system is shown in the sketches below.
We need four quantities to describe the dimensions of a T-beam:

- width of web = \( b_w \)
- flange width = \( b_f \) (derived from beam spacing, beam span length, slab thickness)
- slab thickness = \( t \)
- beam depth = \( h \)

The flange width (\( b_f \)) is not always equal to the beam spacing because the compressive stresses in the flange are not evenly distributed. Due to a phenomenon called "shear lag", the compressive stress in the flange decreases toward the flange ends (see figure below). For design, the total compressive force in the flange is assumed to be distributed uniformly over a flange width called the "effective" flange width. We will use the symbol \( b_f \) for effective flange width in design; (actual flange width is not used for design).
The compression zone in a T-beam is divided into two regions: the web, which we will analyze just as we did for a rectangular beam, and the flange overhang, of width = $b_f - b_w$.

**Important:** The depth of the stress block in the flange overhang cannot be > the slab thickness, $t$. Or: $\alpha_{\text{flange}} = \min(\alpha_{\text{web}}, t)$

**Example.** Analyze two sections of a continuous beam:

1) at midspan of an interior span, and  
2) at the support of an interior span

- **span** = 27 ft  
- $w^{\text{SD}}$ = 10 psf  
- $w^{\text{LL}}$ = 40 psf  
- $f'c$ = 5000 psi  
- $f_y$ = 60,000 psi  
- $t_{\text{stab}}$ = 5 in  
- beam_spacing = 12 ft  
- $h$ = 16 in  
- $b_w$ = 8 in  
- midspan reinforcement: 4 #6 bars in 2 layers at bottom  
- $A_s$ = 1.76 in$^2$  
- Reinforcement at support:  
  2 #6 bars in bottom  
  8 #5 bars in top in one layer

**Midspan of interior span: T-beam**

from spreadsheet: $y_t$ = 0.38 in

**Calc. $M_u$**

$$w^{\text{sw}} = \text{unit wt} \left[ (b_w)(h) + (\text{beam}_\text{spacing} - b_w)(t_{\text{slab}}) \right]$$  
$$w^{\text{sw}} = (0.150\text{kcf}) \left[ (8\text{"})(16\text{"}) + (12' \times 12" - 8\text{"})(5\text{"}) \right] (1\text{ft}^2/144\text{in}^2) = 0.842 \text{klf}$$  

$$w^{\text{SD}} = (w^{\text{SD}})(\text{beam spacing}) = (0.010\text{ksf})(12') = 0.120 \text{klf}$$  

$$w^{\text{LL}} = (w^{\text{LL}})(\text{beam spacing}) = (0.040\text{ksf})(12') = 0.480 \text{klf}$$
\[
\begin{align*}
&w_u = 1.2 \left( w_{sw}^{w} + w_{SD}^{w} \right) + 1.6 \ w_L^{w} \\
&w_u = 1.2 \left( 0.842 \text{ klf} + 0.120 \text{ klf} \right) + 1.6 \ (0.480 \text{ klf}) = 1.922 \text{ klf} \\
&M_u = w_u L^2 / 16 \quad \text{(ACI moment coefficient for 've M in interior span, ACI 8.3)} \\
&= (1.922 \text{ klf}) (27 \text{ ft})^2 / 16 \\
&M_u = 87.6 \text{ k-ft} \\
\end{align*}
\]

Calc. effective flange width, \( b_f \): (ACI 8.10.2)
\[
\begin{align*}
&b_f = \text{minimum of:} \quad \frac{\text{span}}{4} \quad 16 \ t_{slab} + b_w \quad \text{beam spacing} \\
&27'(12'')/4 = 81'' \quad 16(5'') + 8'' = 88'' \quad 12' \times 12' = 144'' \\
&b_f = 81'' \\
\end{align*}
\]

Calc. stress resultants
\[
\begin{align*}
a = \beta y_t = 0.80 \ (0.38 \text{ in}) = 0.307 \text{ in} < 5 \text{ in} = \ t \quad \text{(therefore compression block does not extend below flange)} \\
C_w = 0.85 f'_c a b_w = 0.85 \ (5000 \text{psi})(0.307'')(8'') = 10.4 \text{ k} \\
C_f = 0.85 f'_c a \ (b_f - b_w) = 0.85 \ (5000 \text{psi})(0.307'')(81'' - 8'') = 95.2 \text{ k} \\
T = A_s f_y = (1.76 \text{ in}^2)(60 \text{ ksi}) = 105.6 \text{ k} \quad \text{(Assume steel yields)} \\
\Sigma F_H: \ C = T?, \quad 10.4 \text{ k} + 95.2 \text{ k} = 105.6 \text{ k}, \ OK \\
\end{align*}
\]

Calc. strain in bottom layer of steel (needed to check assumption that steel yields and to calculate \( \phi \)).
\[
\begin{align*}
&\frac{0.003}{y_t} = \frac{0.003 + \varepsilon_s}{d_{\text{bottom layer}}} \\
&d_{\text{bottom layer}} = 16'' - \left[ 1.5'' + \frac{3''}{8} + \frac{1}{2} \right] = 13.75'' \\
&\frac{0.003}{0.38''} = \frac{0.003 + \varepsilon_s}{13.75''}, \quad \varepsilon_s = 0.105 \\
\therefore \quad \text{steel has yeilded} (\varepsilon_s > \varepsilon_y = 0.002) \\
&\phi = 0.90 \ (\varepsilon_s >= 0.005) \\
\end{align*}
\]

Calc. \( \phi M_n \)

Sum moments about neutral axis
\[
\begin{align*}
\phi M_n = \phi \left[ C_w (y_{C_w}) + C_f (y_{C_f}) + T (y_{T}) \right] \\
y_{C_w} = y_t - a/2 = 0.38'' - 0.307'' / 2 = 0.227''
\end{align*}
\]
Flexure Analysis for T-Beams and Compression Reinforcement

\[ y_{C_f} = y_t - a/2 = 0.38" - 0.307" / 2 = 0.227" \]
\[ y_T = d - y_t = 15.0" - 6.02" = 8.98" \]

where

\[ d = 16" - \left[ 1.5" + \frac{3"}{8} + \frac{6"}{8} + \frac{1"}{2} \right] = 12.88", \text{ use } d = 12.75" \text{(round to nearest 1/4")} \]

\[ \phi M_n = 0.90 \left[ 10.4 \text{ k (0.23")} + 95.2 \text{ k (0.23")} + 105.6 \text{ k (12.37")} \right](1' / 12"), \]

\[ \phi M_n = 99.8 \text{ k-ft} > 87.6 \text{ k-ft} = M_u, \text{ OK} \]

**At Interior Side of Interior Support: Compression Steel**

from spreadsheet: \( y_t = 4.17" \)

The bottom of the web is in compression, everything above the neutral axis (\( y_t \) from extreme compression fiber) is cracked. The top steel is spread laterally across the flange (therefore it is always in one layer).

Calc. \( M_u \)

\[ M_u = w_u L^2 / 11 \] (ACI moment coefficient for -'ve M in interior span, ACI 8.3)

\[ = (1.922 \text{ klf}) (27 \text{ ft})^2 / 11 \]

\[ M_u = 127 \text{ k-ft} \]

Calc. stress resultants

\[ a = \beta_1 y_t = 0.80 (4.17 \text{ in}) = 3.34 \text{ in} \]

\[ C_c = 0.85 f_c a b_w = 0.85 (5000 \text{psi})(3.34") (8") = 114 \text{ k} \text{ (compressive force in concrete)} \]
Strain in compression steel = \varepsilon_s'
\frac{0.003}{y_y} = \frac{\varepsilon_s'}{y_y - d'}
\frac{0.003}{4.17''} = \frac{\varepsilon_s'}{4.17'' - 2.25''}, \quad \varepsilon_s' = 0.00138

where \( d' = \text{cover} + \text{diam stirrup} + \frac{\text{diam bar}}{2} = 1.5'' + \frac{3}{8} + \frac{16}{28} = 2.25'' \)

(note: round \( d' \) up to the nearest \( \frac{1}{4}'' \) if necessary)

\( f_s' = \varepsilon_s' \times 29,000 \text{ ksi} = 0.00138 \times 29,000 \text{ ksi} = 40.0 \text{ ksi} \quad (< f_y = 60 \text{ ksi}, \text{ so use } f_s' = 40 \text{ ksi})

\( C_s = A_s' \times f_s' = (0.88 \text{ in}^2) (40 \text{ ksi}) = 35.3 \text{ k} \)

\( T = A_s f_y = (2.48 \text{ in}^2)(60 \text{ ksi}) = 148.8 \text{ k} \quad \text{(Assume steel yields)}

\( \Sigma F_H: C = T?, \quad C_c + C_s = T?, \quad 114 \text{ k} + 35 \text{ k} = 149 \text{ k}, \text{ OK} \)

Calc. strain in tension steel (needed to check assumption that steel yields and to calculate \( \phi \)).
\frac{0.003}{y_s} = \frac{0.003 + \varepsilon_s}{d}
\quad d = 16'' - [1.5'' + \frac{3''}{8} + \frac{15}{28}] = 13.81'', \text{ use } d = 13.75''
\frac{0.003}{13.75''} = \frac{0.003 + \varepsilon_s}{13.75''}, \quad \varepsilon_s = 0.00689

\therefore \text{ steel has yeilded } (\varepsilon_s > \varepsilon_y = 0.002)

& \quad \phi = 0.90 \ (\varepsilon_s >= 0.005)

Calc. \( \phi M_n \)

Sum moments about neutral axis
\( \phi M_n = \phi \left[ C_c (y_{C_c}) + C_s (y_{C_s}) + T (y_T) \right] \)
\( y_{C_c} = y_t - a/2 = 4.17'' - 3.34'' / 2 = 2.50'' \)
\( y_{C_s} = y_t - d' = 4.17'' - 2.25'' = 1.92'' \)
\( y_T = d - y_t = 13.75'' - 4.17'' = 9.58'' \)

\( \phi M_n = 0.90 \left[ 114 \text{ k} (2.50'') + 35 \text{ k} (1.92'') + 149 \text{ k} (9.58'') \right] (1'' / 12''), \quad \phi M_n = 133 \text{ k-ft} \)

\( \phi M_n = 133 \text{ k-ft} > 127 \text{ k-ft} = M_u, \text{ OK} \)