Another principal failure mode of reinforced concrete components, after flexure, is shear. Flexure cracks form where the flexural tension stresses are greatest, for example at the bottom of the midspan segment of a simply supported beam. Cracks due to shear forces form where the tension stresses due to shear are greatest, for example near the supports of a simply supported beam.

The basic design equation for shear says that the reduced nominal shear capacity must be greater than the factored shear force.

\[ \phi V_n > V_u \]  \hspace{1cm} (11-1)

The strength reduction factor for shear = 0.75 \hspace{1cm} (9.3.2.3)

**Shear Strength of "Concrete".** The nominal shear strength \( V_n \) is composed of the sum of the nominal shear strength provided by "concrete" \( V_c \) and the nominal shear strength provided by shear reinforcement \( V_s \). The ultimate nominal shear strength provided by concrete \( V_c \) is actually provided by several mechanisms, illustrated in the figure above, including:

- shear strength of concrete in compression zone,
- the vertical component of aggregate interlock, and
- dowel action of the flexural reinforcement.

The ACI code provides two equations for calculating \( V_c \) but we will only use one. This is what the author of your text recommends and also it makes shear design much simpler.

\[ V_c = 2\sqrt{f'_c b_w d} \]  \hspace{1cm} (11-3)
Shear Reinforcement. Shear reinforcement is oriented perpendicular to the flexural reinforcement. Shear reinforcement can take the form of stirrups (typically used for beams), welded-wire fabric (used for joists), and ties/spiral cages for columns.

Shear reinforcement keeps cracks parallel to the flexural reinforcement small. Shear reinforcement that encloses the core of the member (all but WWF) serves to confine the concrete. Confining the concrete in the core has several effects at ultimate strength conditions (concrete is cracked):

- Friction between pieces of concrete is increased; e.g. aggregate interlock increases beam shear strength.
- Vertical load capacity of columns is increased by horizontal confinement of concrete by ties and especially by spiral cages.
- Splitting failure on a horizontal plane through the flexural reinforcement of a beam is reduced.

In a shear failure, the bottom portion of a beam wants to separate from the top. To be effective, shear reinforcement should be anchored on each end. This can be accomplished by wrapping the stirrup around the longitudinal (flexural) reinforcement on the top and bottom (stirrups) or welding the vertical members to longitudinal bars (WWF). Also, as much of the shear reinforcement as possible should extend into the compression zone. For these reasons, shear reinforcement should extend as close to the top and bottom surface of the beams as cover will allow.
Shear Strength of Shear Reinforcement. The shear strength provided by the shear reinforcement \( (V_s) \) at a section is calculated as follows.

\[
V_s = A_v f_y n
\]

\( A_v \) = cross-section area of one stirrup (both legs)

\( n \) = number stirrups crossing crack

We assume that the diagonal tension crack is at \( 45^\circ \) from the vertical. Then the horizontal projected length of the crack = \( d \).

So the number of stirrups crossing the crack is:

\[
n = \frac{d}{s}, \text{ where } s = \text{stirrup spacing}
\]

\[
V_s = A_v f_y \frac{d}{s} \quad \text{(ACI 11-15)}
\]

Limits on Shear Reinforcement. The ACI sets several limits regarding shear reinforcement to ensure that

1. sufficient shear reinforcement is provided to prevent shear failure (sufficient \( V_s \)), and
2. the shear reinforcement is distributed so that at a sufficient number of bars cross the bottom-half of the shear crack (\( n \) bars every \( d/2 \)). The horizontal projection of the shear crack from bottom of beam to mid-height is approximately \( d/2 \).

The designer meets the ACI criteria by specifying two parameters:

- stirrup size (represented in Equation 11-15 above by the cross-sectional area of the stirrup crossing the shear crack, \( A_v \))
- stirrup spacing \( (s) \)

<table>
<thead>
<tr>
<th>ACI Criteria</th>
<th>Design Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_s )</td>
<td>Stirrup Size ( (A_v) )</td>
</tr>
<tr>
<td>(1) ( V_s )</td>
<td>is increased by ↑</td>
</tr>
<tr>
<td>(2) ( n ) bars per ( d/2 )</td>
<td>is increased by</td>
</tr>
</tbody>
</table>

As indicated in the table above, both stirrup size and spacing affect the first criteria (sufficient \( V_s \)), but only stirrup spacing affects the second criteria (sufficient number of
stirrups crossing the shear crack). Also, decreasing the spacing improves both the shear strength ($V_s$) and the number of stirrups crossing the shear crack (n bars per d/2).

Other stirrup design criteria (not specified by ACI) are constructability and economy. Stirrups should not be placed closer than 3" or 4" apart so that the contractor can place the concrete between the stirrups. Also, specifying significantly more stirrups than are needed should be avoided.

**Stirrup Design Equations**

Stirrup design typically involves selecting a stirrup size (#3 or #4 are common), and determining the largest spacing that will meet criteria (1) and (2). As the factored shear force ($V_u$) increases relative to the concrete shear strength ($V_c$), the criteria become more stringent resulting in smaller maximum allowed stirrup spacing. A summary sheet of shear design equations presents the maximum allowed stirrup spacing for each range of $V_u / V_c$.

The ACI specifications are presented in the following sections. In each section, the ACI equations are presented and explained, and then rearranged to produce the equations in the Shear Design Equations summary sheet. The first three specifications concern criteria (1) $V_s$, and the next two specifications concern criteria (2) $n / d$.

1. **Minimum Shear Reinforcement.** ACI 11.5.6.1 requires that a minimum amount of shear reinforcement be provided if the factored shear is greater than half the shear capacity of the concrete.

   \[
   \text{min. shear reinforcement is required if } V_u > \frac{\phi V_c}{2}, \quad \text{or}
   \]

   \[
   \text{when } \frac{V_u}{\phi V_c} > \frac{1}{2}
   \]

   Shear failures are sudden and the predicted shear strength of a component is highly variable. The minimum shear reinforcement must provide a minimum shear force per unit area $(v_s = \frac{V_s}{b_n d})$ of $0.75 \sqrt{f'_c}$ or 50psi, whichever is greater.

   \[
   \min v_s = \max[0.75 \sqrt{f'_c}, 50 \text{psi}]
   \]

   The above equation can be expanded and then rearranged to solve for spacing (s)

   \[
   \max s = \frac{A_s f_y d}{\max[0.75 \sqrt{f'_c}, 50 \text{psi}] b_n d}
   \]
2. **Vs of Shear Reinforcement.** ACI 11.4.61 specifies that when \( V_u \) exceeds \( \phi V_c \), then \( V_s \) shall be provided so that

\[
\phi (V_c + V_s) \geq V_u \quad \text{(ACI 11-1, 11-2), or}
\]

ACI Section 11.4.7.2 specifies \( V_s \) (derived above)

\[
V_s = A_s f_y \frac{d}{s} \text{ or }
\]

when \( \frac{V_u}{\phi V_c} > 1 \)

\[
\max s = \frac{A_s f_y d}{V_u - V_c}
\]

3. **Maximum Shear Reinforcement.** ACI 11.4.7.9 also requires that the shear reinforcement provide a shear force per unit area \( (v_s) \) of no more than \( 8\sqrt{f'c} \) This limits the stress in the stirrups and thereby prevents the diagonal cracks from growing too large.

\[
\max v_s = 8\sqrt{f'c}
\]

The equation above can be expanded and rearranged as follows:

\[
v_s = \frac{V_s}{b_s d}, \text{ so } \max v_s = 8\sqrt{f'c} b_s d
\]

\[
also \quad V_s = \frac{V_u}{\phi} - V_c, \text{ so } \max \left[ \frac{V_u}{\phi} - V_c \right] = 8\sqrt{f'c} b_s d
\]

\[
and \quad V_c = 2\sqrt{f'} b_s d,
\]

\[
\therefore \max \left[ \frac{V_u}{\phi} - V_c \right] = 4V_c, \text{ or } \max \left[ \frac{V_u}{\phi} \right] = 5V_c, \text{ or }
\]

\[
\max \left[ \frac{V_u}{\phi} \right] = 5
\]

4. **Minimum Number of Stirrups Crossing Bottom-Half of Diagonal Crack.** ACI 11.4.5.1 requires that stirrups be placed so that at least one stirrup crosses the shear crack in the bottom-half of the beam (1 bar per \( d/2 \)). This corresponds to a maximum spacing of \( d/2 \).

\[
\max s = \min \left[ \frac{d}{2}, 24" \right]
\]

5. **Minimum Number of Stirrups when \( V_s > 4\sqrt{f'c} b_s d \).** ACI 11.4.5.3 specifies that when the stirrups must carry large forces \( (V_s > 4\sqrt{f'c} b_s d) \), the maximum allowable stirrup spacing should be decreased by a factor of 2.

\[
\max s = \min \left[ \frac{d}{2}, 24" \right]
\]
Design of Beams for Shear.

Designing a beam to resist shear is a straightforward process:

- Calculate the uniform distributed loads \( w^D, w^L \) and \( w_u \)
- Calculate the envelope of factored shear \( V_u \), or to make using the shear design equations summary sheet easier, calculate the envelope of \( \frac{V_u}{\phi} \).
- Start at the critical section\(^2\) and design the stirrups
- Design the stirrups at midspan
- Design the stirrups at a intermediate section between the critical section and midspan.

Determine the location of the transition points (distances from face of support) between:
- the critical section and the intermediate section, and
- the intermediate section and midspan

---

\( V_s = \frac{V_u}{\phi} - V_c \) and \( 4\sqrt{f_c} b_u d = 2 V_c \) are substituted into the inequality above it becomes

\[
\frac{V_u}{\phi} - V_c > 2 V_c, \quad \text{or} \quad \frac{V_u}{\phi} > 3 V_c
\]

therefore

\[
\frac{V_u}{\phi} > 3 V_c \Rightarrow \max s = \min \left[ \frac{d}{4}, 12'' \right]
\]

---

\(^1\) The maximum shear at the support is specified in ACI 8.3.3. The maximum shear at midspan is obtained by applying the live load to half the span.

\(^2\) Critical Section. A diagonal tension crack extending from beneath the edge of support terminates at a projected horizontal distance equal to \( d \) from the face of the support. A diagonal tension crack forming closer to the support will terminate within the support, and will not result in failure. ACI identifies the section located a distance \( d \) from the face of the support as the "critical section", and allows the factored shear between the face of the support and the critical section to be equal to the shear at the critical section.
Design of Beams for Shear

- Critical section: $V_u$ @ face of support, due to factored shear envelope
- Critical section: $V_u$ @ midspan, due to midspan