Design a one-way slab for an interior bay of a multi-story office building using the information specified below. Neglect compression reinforcement. Assume partitions cannot be damaged by deflections. Round slab thickness up to the nearest $\frac{1}{4}”$.

Use a spreadsheet to design each section and show your results with a sketch (elevation view). Submit hand calculations matching the spreadsheet for one section. Include units on all numbers.

Plan View of Floor System

\[
f’c = 5000 \text{ psi} \quad w^S = 10 \text{ psf} \quad \text{beam spacing} = 18 \text{ ft}
\]

\[
f_y = 60,000 \text{ psi} \quad w^L = 100 \text{ psf} \quad b_w \text{ of beam} = 14 \text{ in}
\]
Slab Design Example

2. Design Flexure Reinforcement at each Section

Min. Reinforcement:

\[ \frac{A_{s_{\min}}}{bh} = 0.0018 \quad \text{(ACI 10.5.4)} \]

\[ A_{s_{\min}} = (0.0018)(12\text{"})(7.75\text{"}), \]

\[ A_{s_{\min}} = 0.167 \frac{\text{in}^2}{\text{ft width}} \]

Max. Reinforcement Spacing (ACI 10.5.4):

\[ S_{\text{max, slab}} = \min[18\text{in}, 3h] = \min[18\text{in}, 3(7.75\text{in})] = 18\text{in} \]
**Slab Design Example**

Distribution of reinforcement (ACI 10.6.4):

\[ S_{\text{max, distrib}} = \min \left[ 15 \frac{40,000}{2} - 2.5c_c, 12 \frac{40,000}{2} \right] \]

\[ c_c = h - d - \frac{\phi_{\text{bar}}}{2} = 7.75\text{in} - 6.75\text{in} - \frac{1}{2} = 0.75\text{in} \]

\[ S_{\text{max, distrib}} = \min \left[ 15 \frac{40,000}{2} - 2.5 \left( 0.75\text{in} \right), 12 \frac{40,000}{2} \left( 60,000\text{ psi} \right) \right] \]

\[ S_{\text{max}} = \min \left[ S_{\text{max, slab}}, S_{\text{max, distrib}} \right] = \min \left[ 18\text{in}, 12\text{in} \right], \quad S_{\text{max}} = 12\text{in} \]

**Design of Bar Spacings:**

![Elevation View of Floor Slab](image)

We can design one half of the slab length due to symmetry.

**Design Sections**

<table>
<thead>
<tr>
<th>M Coeff. (ACI 8.3)</th>
<th>1/24</th>
<th>1/14</th>
<th>1/10</th>
<th>1/11</th>
<th>1/16</th>
<th>1/11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mu, k-ft</td>
<td>3.40</td>
<td>5.83</td>
<td>8.17</td>
<td>7.43</td>
<td>5.10</td>
<td>7.43</td>
</tr>
<tr>
<td>As, req'd, in²</td>
<td>0.118</td>
<td>0.202</td>
<td>0.283</td>
<td>0.257</td>
<td>0.177</td>
<td>0.257</td>
</tr>
<tr>
<td>s, in</td>
<td>12</td>
<td>11.87</td>
<td>8.48</td>
<td>9.33</td>
<td>13.57</td>
<td>9.33</td>
</tr>
<tr>
<td>As, prov'd, in²</td>
<td>0.20</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>r, in</td>
<td>0.235</td>
<td>0.27</td>
<td>0.30</td>
<td>0.30</td>
<td>0.20</td>
<td>0.30</td>
</tr>
<tr>
<td>εs</td>
<td>0.0659</td>
<td>0.314</td>
<td>0.353</td>
<td>0.353</td>
<td>0.235</td>
<td>0.353</td>
</tr>
<tr>
<td>φMn, k-ft</td>
<td>5.97</td>
<td>0.0486</td>
<td>0.0429</td>
<td>0.0429</td>
<td>0.0659</td>
<td>0.0429</td>
</tr>
</tbody>
</table>

Calculations for Column 2 provided below. Calculations for Columns 3 through 7 performed by spreadsheet (attached)
Typical calcs. for first column of above table:

\[ M_u = \frac{w}{24} \left( \frac{16.83 \text{ ft}}{24} \right)^2 = 3.40 \text{ k-ft} \]

\[ A_{s_{\text{req'd}}} \]

Set \( \varphi M_n = M_u \)

\( \varphi A_s f_y (d - \frac{a}{2}) = M_u \)  \( \text{(Assume steel yields at ultimate strength)} \)

Assume: \( d - \frac{a}{2} \approx 0.95d \)

\[ A_{s_{\text{req'd}}} = \frac{M_u}{\varphi f_y 0.95d} = \frac{(3.40 \text{k-ft})(12 \text{ in})}{(0.90)(60 \text{ksi})(0.95)(6.75 \text{in})} = 0.118 \text{ in}^2/\text{ft-width} < A_{s_{\text{min}}} \]

\( \therefore A_{s_{\text{req'd}}} = 0.167 \text{ in}^2/\text{ft width} \)

Rebar spacing, \( s \):

\[ s = \frac{A_{\text{bar}}}{A_{s_{\text{req'd}}}} = \frac{0.20 \text{ in}^2}{0.167 \text{ in}^2/\text{12" width}} = 14.4 \text{ in} > S_{\text{max}} = 12 \text{ in} \]

\( \therefore s = 12 \text{ in} \)

Strain in Reinforcement, \( \varepsilon_s \)

\[ A_{s_{\text{prov'd}}} = A_{\text{bar}} \frac{12}{s} = \frac{(0.20 \text{ in}^2) (1 \text{ bar})(12 \text{ in})}{(0.167 \text{ in}^2/\text{12" width})} = 0.20 \text{ in}^2/\text{ft-width} \]

From C = T:

\[ 0.85 f_y a b = A_s f_y \]

\[ 0.85(5 \text{ksi}) a (12 \text{in}) = (0.20 \text{in}^2)(60 \text{ksi}) \quad a = 0.235 \text{in} \]

\[ y_i = \frac{a}{\beta_i} = \frac{0.235 \text{in}}{0.80} = 0.294 \text{in} \]

\[ 0.003 = 0.003 + \varepsilon_s \quad 0.003 = 0.003 + \varepsilon_s \quad \frac{0.003 + \varepsilon_s}{d} = \frac{0.003 + \varepsilon_s}{6.75 \text{in}} \]

\[ \varepsilon_s = 0.0658 \]

\( \therefore \varepsilon_s > 0.004 = \text{ACI min. for beams and slabs, OK} \)

\[ \varepsilon_s > 0.005 \text{ so } \phi = 0.90 \text{ as assumed, OK} \]

\( \phi M_n \):

\[ \phi M_n = \phi A_s f_y (d - \frac{a}{2}) = (0.90)(0.20 \text{in}^2)(60 \text{ksi})(6.75 \text{in} - \frac{0.235 \text{in}}{2}) \frac{1 \text{ft}}{12 \text{in}} \]

\( \phi M_n = 5.97 \text{k-ft} \)
Design bar cutoffs:
Extension of top bars past face of support (from Text Figure A-5c) =

\[ \frac{l_n}{4} = \frac{176''}{4} = 44'' = 3'8'' \text{ at end beams} \]

\[ 0.3l_n = 0.3(176'') = 53'' = 4'5'' \text{ at interior beams} \]

Temperature & Shrinkage Steel:

Use #4 bars (not bent by workmen walking on)

Also, \( A_{s, req'd} \) for T&S reinforcement = \( A_{s, min} \) for slabs = 0.167 \( \frac{in^2}{ft - width} \)

\[ s = \frac{A_{bar}}{A_{s, req'd}} = \frac{0.20in^2}{0.167in^2} = 14.4in \]

\[ s_{max} = \min[18'', 5'h] = \min[18in, 5(7.75in)] = 18in \]

T & S Reinf : #4bars @ 12in

\[ \#4\text{ }\text{@ }12'' \quad 3'8'' \quad \frac{\#4}{\text{@ }8''} \quad 4'5'' \quad \text{clear cover } = 0.75'' \text{ top, bottom and sides} \]

\[ \#4\text{ }\text{@ }9'' \quad 4'5'' \quad 7.75'' \quad \#4\text{ }\text{@ }12'' \]

6'' min (Typ.)

\( \text{Slab Details} \)
**Example Slab Design**

### Material Properties
- **Compressive strength of concrete** $f'_c$ = 5,000 psi
- **Coefficient for depth of stress block** $\beta_1$ = 0.80
- **Yield strength of reinforcement** $f_y$ = 60,000 psi
- **Unit weight of reinforcement** $\text{UW}$ = 150 pcf
- **Modulus of elasticity of concrete** $E_c$ = 4,030,000 psi
- **Modulus of rupture of concrete** $f_r$ = 530 psi
- **Modulus of elasticity of steel** $E_s$ = 29,000,000 psi

### Geometry
- **Span length of slab (beam spacing)** $L$ = 18 ft
- **Width of supporting beam** $b_{support}$ = 14 in
- **Clear span** $L_n$ = 16.83 ft
- **Minimum thickness** $h_{min}$ = 7.714 in
- **Depth of slab** $h$ = 7.75 in

### Loads
- **Self weight of beam** $s_w$ = 0.0969 klf
- **Superimposed dead load** $\text{SDL}$ = 10 psf
- **Live load** $\text{LL}$ = 100 psf
- **Uniform distributed factored load** $\omega_u$ = 0.288 klf
  $$\omega_u = 1.2 * (s_w + \text{SDL}/1000 * 1) + 1.6 * \text{LL}/1000 * 1$$

### Reinforcement
- **Size of flexural reinforcement** $\Phi_{\text{bar}}$ = 0.5 in
- **Area of bar** $A_{\text{bar}}$ = 0.20 in^2
- **Clear cover** $\text{cover}$ = 0.75 in
- **Maximum effective depth** $d_{\text{max}}$ = 6.75 in

### Shear
- **OK** = IF($\phi V_n > V_u$, "OK", "NG")
- **Max shear force** $V_u$ = 2.79 k = $1.15 * \omega_u * L_n/2$
- **Shear strength of concrete** $\phi V_n$ = 8.59 k = $0.75 * 2 * \sqrt{f'_c} * d / 1000$

### Flexure
- **Min. reinforcement** $A_{s, \text{min}}$ = 0.167 in^2 = 0.0018 * 12 * $h$
- **Max spacing for slabs** $s_{\text{max, slabs}}$ = 18.00 in = MIN(18, 3 * $h$)
- **Clear distance from reinf. to ten. face** $c_c$ = 0.750 in = $h - d - \Phi_{\text{bar}}/2$
- **Max spacing for distribution** $s_{\text{max, distrib}}$ = 12.00 in
- **Max spacing** $s_{\text{max}}$ = 12.00 in = MIN($s_{\text{max, slabs}} - s_{\text{max, distrib}}$)
Example Slab Design

<table>
<thead>
<tr>
<th>coefficient from ACI 8.3</th>
<th>Mcoef</th>
<th>24</th>
<th>14</th>
<th>10</th>
<th>11</th>
<th>16</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>moment due to factored loads</td>
<td>M_u</td>
<td>3.40</td>
<td>5.83</td>
<td>8.17</td>
<td>7.43</td>
<td>5.10</td>
<td>7.43 k-ft</td>
</tr>
<tr>
<td>As_req'd for ( \phi ) Mn to equal Mu</td>
<td>A_s_req'd</td>
<td>0.167</td>
<td>0.202</td>
<td>0.283</td>
<td>0.257</td>
<td>0.177</td>
<td>0.257 in^2</td>
</tr>
<tr>
<td>s req'd for #4 bar to provide As_req'd</td>
<td>s_req'd</td>
<td>12.00</td>
<td>11.87</td>
<td>8.48</td>
<td>9.33</td>
<td>13.57</td>
<td>9.33 in</td>
</tr>
<tr>
<td>design bar spacing</td>
<td>s</td>
<td>12</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>As for #4 bar at spacing &quot;s&quot;</td>
<td>A_s_prov'd</td>
<td>0.20</td>
<td>0.27</td>
<td>0.30</td>
<td>0.30</td>
<td>0.20</td>
<td>0.30 in^2</td>
</tr>
<tr>
<td>depth of stress block</td>
<td>a</td>
<td>0.235</td>
<td>0.314</td>
<td>0.353</td>
<td>0.353</td>
<td>0.235</td>
<td>0.353 in</td>
</tr>
<tr>
<td>strain in steel reinforcement at ult.</td>
<td>( \varepsilon_s )</td>
<td>0.0659</td>
<td>0.0486</td>
<td>0.0429</td>
<td>0.0429</td>
<td>0.0659</td>
<td>0.0429</td>
</tr>
<tr>
<td>strength reduction factor</td>
<td>( \phi )</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>available flexure strength</td>
<td>( \phi M_n )</td>
<td>5.97</td>
<td>7.91</td>
<td>8.87</td>
<td>8.87</td>
<td>5.97</td>
<td>8.87 k-ft</td>
</tr>
</tbody>
</table>

where

\[
M_u = \varepsilon_u * L_e^2 / Mcoef
\]

\[
A_{s, req'd} = \text{MAX}(A_{s, min}, M_u * 12000 / (0.9 * f_y * 0.95 * d))
\]

\[
s_{req'd} = \text{MIN}(s_{max}, A_{bar} / (A_{s, req'd} / 12))
\]

\[
a = A_{s, prov'd} * f_y / (0.85 * f_c * 12)
\]

\[
\varepsilon_s = 0.003 / (a / \beta_1) * d - 0.003
\]

\[
\phi = \text{IF}(\varepsilon_s < 0.002, 0.65, \text{IF}(\varepsilon_s > 0.005, 0.9, 0.48 + 83 * \varepsilon_s))
\]

\[
\phi M_n = \phi * A_{s, prov'd} * f_y * (d - a/2) / 12000
\]