These notes present the procedures for analyzing beams. Beams are members loaded transversely subject to flexure and shear. Chapters F and G of the Specifications, pg 16.1-44 – 69. Members subject to combined flexure and compression will be covered later (Chapter H in the Specifications). These notes will focus on I-shaped members.

**Failure Modes**
As with compression members, members in flexure can fail in three different ways:

1. **Material Failure (Yielding)** leading to a plastic hinge,
2. **Stability Failure** over the length of the member called lateral-torsional buckling, and
3. **Stability Failure** of a plate-shaped cross-section element of the member (local buckling).

1. **Plastic Hinge.** When normal stresses due to flexure exceed the yield stress at all points in the beam cross-section, a plastic hinge is said to form. Although the load and associated bending moment required to just form the plastic hinge can still be supported, additional bending moment will cause unlimited deformation at the hinge, leading to redistribution of load for indeterminate structures, and collapse for determinate structures such as the simply-supported beam in Figure 1.

![Figure 1. Formation of a plastic hinge](image)

2. **Lateral-torsional Buckling.** Flexure causes compressive and tensile normal stresses. For example, gravity loads on a simply-supported beam will cause compression in the top of the beam, and tension in the bottom. The compression in the top of the beam can cause the top of the beam to buckle laterally. Since the bottom of the beam is in tension, it does not buckle, causing the beam to twist about its longitudinal axis as the top buckles laterally (see Figure 2). Lateral-torsional buckling can be prevented by bracing the beam so that it cannot deflect laterally or twist.
3. Local Buckling. As with compression members, local buckling of the plate-shaped elements of the cross-section may occur. Depending on the slenderness of the plate-shaped element (width to thickness ratio, b/t), the cross section of a flexure member is classified as either compact, non-compact, or slender.

- Compact sections can develop the full plastic moment without buckling. Most W-shape sections are compact.
- Plate-shaped elements of non-compact sections will buckle inelastically before the plastic moment is reached.
- Plate-shaped elements of slender sections will buckle elastically before the plastic moment is reached.

Design Equations

1. Plastic Moment Capacity, $M_p$. Normal stresses due to flexure are distributed linearly over the depth the cross section, as illustrated in Figure 1a below. Increased flexure causes the outer “fibers” (furthest from the neutral axis) to yield, as indicated in Figure 1b. Ultimately, all parts of the cross-section yield (Figure 1c).
The moment corresponding to Figure 1c is called the “plastic moment”, $M_p$. It can be calculated by summing forces about the neutral axis (see Figure 2).

$$ M_p = \int \sigma_y \cdot y \, dA = \left[ \sigma_y \cdot \frac{A}{2} \cdot \bar{y} \right] + \left[ \sigma_y \cdot \frac{A}{2} \cdot \bar{y} \right] $$

$$ M_p = \sigma_y \cdot A \cdot \bar{y} $$

where $A$ is the gross area of the section and $\bar{y}$ is the distance from the neutral axis to the centroid of the half-section.
Figure 2. Compressive and tensile stresses on section corresponding to plastic moment.

Using AISC terminology, the plastic moment, $M_p$ is

$$M_p = F_y Z_x$$  \hspace{1cm} \text{Eqn. F2-1}

Where $Z_x$ is called the plastic section modulus.

2. Lateral Torsional Buckling (LTB). Lateral torsional buckling is a function of the unbraced length, $L_b$. The unbraced length is the distance between lateral supports to the compression flange. In Figure 3 below, the center beam has three unbraced segments with $L_b = 10', 15'$ and 10', respectively.

Figure 3. Unbraced lengths ($L_b$) for lateral torsional buckling.
The AISC equations for the nominal flexure strength, $M_n$, for a beam subject to lateral torsional buckling are summarized in Figure 4 below. $M_n$ depends on the unbraced length, $L_b$. If $L_b$ is less than $L_p$ (p for plastic), then the full plastic moment ($M_p$) can be developed. If $L_b$ is greater than $L_r$, then the beam will fail in elastic lateral torsional buckling. If $L_b$ is between $L_p$ and $L_r$, then the beam will fail in inelastic lateral torsional buckling.

$$L_p = 1.76r_y \frac{E}{F_y} \quad L_r = 1.95r_y \frac{E}{0.7F_y} \sqrt{\frac{J(1)}{S_x h_0}} \left[1 + \sqrt{1 + 6.76 \left(\frac{0.7F_y S_x h_0}{E J(1)}\right)^2}\right]$$

![Figure 4](image)

**Figure 4.** Design equations for Lateral Torsional Buckling (LTB)

The equations for nominal flexure strength ($M_n$) with LTB were derived assuming a constant magnitude bending moment distribution along the beam. $M_n$ for unbraced beam segments with non-uniform bending moment diagrams will be higher. This is accounted for with the $C_b$ factor, which is similar to the effective length factor ($k$) for axial buckling (see Sect. F2, pp. 47-48). Values for $C_b$ for common lateral brace configurations are given in Table 3-1 (pg 3-18) in the Manual. For beams with uniform loads, $C_b$ can usually be conservatively yet reasonably assumed to be equal to 1.0.

3. **Compression Flange Local Buckling (FLB).** Local buckling of the compression flange (FLB) occurs if the flange is so slender that it buckles before the plastic moment can be developed. The AISC equations for $M_n$ with FLB are summarized in Figure 5 below. $M_n$
depends on the slenderness of the flange ($\lambda = \frac{b_f}{2t_f}$). If $\lambda$ is less than $\lambda_{pf}$, then the full plastic moment can be developed and the section is considered to be compact. If $\lambda$ is greater than $\lambda_{rt}$, then the flange will buckle elastically and the section is considered to be slender. If $\lambda$ is between $\lambda_{pf}$ and $\lambda_{rt}$, then the flange will buckle inelastically and the section is considered to be noncompact.

\[
\lambda_{pf} = 0.38 \sqrt{\frac{E}{F_y}} \quad \lambda_{rt} = 1.0 \sqrt{\frac{E}{F_y}}
\]

\[
\lambda = \frac{b_f}{2t_f}
\]

**Figure 5.** Design equations for $M_n$ with compression flange local buckling.

4. **Shear Failure.** In addition to flexure failure, a beam can fail in shear. The web of a wide-flange beam carries almost all of the shear. The AISC equations for shear strength, $V_n$, are summarized in Figure 6 below.

$V_n$ is a function of the web slenderness, $\frac{h}{t_w}$. If $\frac{h}{t_w} < TP2$ in Figure 6, then the web fails by yielding. The area considered to resist shear is called $A_w$ and is calculated as the beam depth ($d$) times the web thickness ($t_w$). Shear yield strength is equal to 60% of the tensile yield strength ($F_y$). $C_v$ is a coefficient that depends on the type of shear failure, which in
turn depends on the web slenderness, \( \frac{h}{t_w} \). If \( \frac{h}{t_w} > TP3 \) in Figure 6, then the web will buckle elastically. If \( \frac{h}{t_w} \) is between TP2 and TP3, then the web will buckle inelastically.

**Figure 6.** Design equations for Shear Strength

\[
\begin{align*}
V_n &= 0.6 F_y A_w C_v \\
A_w &= d t_w \\
C_v &= \frac{1.10 \sqrt{\frac{k_r E}{F_y}}}{\frac{h}{t_w}} \\
C_v &= \frac{1.51 E k_r}{\left(\frac{h}{t_w}\right)^2 F_y} \\
TP1 &= 2.24 \sqrt[6]{\frac{E}{F_y}} \\
TP2 &= 1.10 \sqrt[6]{\frac{k_r E}{F_y}} \\
TP3 &= 1.37 \sqrt[6]{\frac{k_r E}{F_y}}
\end{align*}
\]

5. **Service-Load Deflection.** Another design criteria in addition to preventing flexure and shear failures, is deflection due to service (unfactored) loads. A beam may have sufficient strength, but if it is too flexible, then non-structural elements such as partitions and ceilings may be damaged. Also, flexible beams may feel too “bouncy” to a building’s occupants. Maximum allowable live load deflections are listed in the International Building Code (IBC), and summarized in Figure 7 below.
<table>
<thead>
<tr>
<th>CONSTRUCTION</th>
<th>L</th>
<th>S or W</th>
<th>D + L^a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof members:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Supporting plaster ceiling</td>
<td>1/360</td>
<td>1/360</td>
<td>1/240</td>
</tr>
<tr>
<td>Supporting nonplaster ceiling</td>
<td>1/240</td>
<td>1/240</td>
<td>1/180</td>
</tr>
<tr>
<td>Not supporting ceiling</td>
<td>1/180</td>
<td>1/180</td>
<td>1/120</td>
</tr>
<tr>
<td>Floor members</td>
<td>1/360</td>
<td>—</td>
<td>1/240</td>
</tr>
<tr>
<td>Exterior walls and interior partitions:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With brittle finishes</td>
<td>—</td>
<td>1/240</td>
<td>—</td>
</tr>
<tr>
<td>With flexible finishes</td>
<td>—</td>
<td>1/120</td>
<td>—</td>
</tr>
<tr>
<td>Farm buildings</td>
<td>—</td>
<td>—</td>
<td>1/180</td>
</tr>
<tr>
<td>Greenhouses</td>
<td>—</td>
<td>—</td>
<td>1/120</td>
</tr>
</tbody>
</table>

For SI: 1 foot = 304.8 mm.

a. For structural roofing and siding made of formed metal sheets, the total load deflection shall not exceed 1/60. For secondary roof structural members supporting formed metal roofing, the live load deflection shall not exceed 1/150. For secondary wall members supporting formed metal siding, the design wind load deflection shall not exceed 1/90. For roofs, this exception only applies when the metal sheets have no roof covering.

b. Interior partitions not exceeding 6 feet in height and flexible, folding and portable partitions are not governed by the provisions of this section. The deflection criterion for interior partitions is based on the horizontal load defined in Section 1607.13.

c. See Section 2403 for glass supports.

d. For wood structural members having a moisture content of less than 16 percent at time of installation and used under dry conditions, the deflection resulting from $L + 0.5D$ is permitted to be substituted for the deflection resulting from $L + D$.

e. The above deflections do not ensure against ponding. Roofs that do not have sufficient slope or camber to assure adequate drainage shall be investigated for ponding. See Section 1611 for rain and ponding requirements and Section 1503.4 for roof drainage requirements.

f. The wind load is permitted to be taken as 0.7 times the "component and cladding" loads for the purpose of determining deflection limits herein.

g. For steel structural members, the dead load shall be taken as zero.

h. For aluminum structural members or aluminum panels used in skylights and sloped glazing framing, roofs or walls of sunroom additions or patio covers, not supporting edge of glass or aluminum sandwich panels, the total load deflection shall not exceed 1/100. For aluminum sandwich panels used in roofs or walls of sunroom additions or patio covers, the total load deflection shall not exceed 1/120.

i. For cantilever members, l shall be taken as twice the length of the cantilever.

Figure 7. Deflection limits from IBC 2006.