Axial forces and bending moments both create normal stresses on a member’s cross section. The combined effect of axial compression and flexure is considered in the AISC Equations H1-1a and H1-1b on Pg. 16.1-73.

\[
\text{for } \frac{P}{P_c} \geq 0.2 : \\
\frac{P}{P_c} + \frac{8}{9} \frac{M_c}{M_c} \leq 1.0 \\
\text{(H1-1a)}
\]

\[
\text{for } \frac{P}{P_c} < 0.2 : \\
\frac{P}{2P_c} + \frac{M_c}{M_c} \leq 1.0 \\
\text{(H1-1b)}
\]

The equations above have been modified from the AISC equations in the Manual to consider flexure about only one axis. The AISC equations in the Manual consider flexure about two axes (the x-axis and the y-axis).

\( P \) and \( M \) represent required strengths, \( P_u \) and \( M_u \) for the LRFD method. \( P_c \) and \( M_c \) represent the design strengths, \( \phi_c P_n \) and \( \phi_b M_n \) for the LRFD method. Using LRFD notation, the equations become

\[
\text{for } \frac{P}{\phi_c P_n} \geq 0.2 : \\
\frac{P}{\phi_c P_n} + \frac{8}{9} \frac{M}{\phi_b M_n} \leq 1.0 \\
\text{(H1-1a)}
\]

\[
\text{for } \frac{P}{\phi_c P_n} < 0.2 : \\
\frac{P}{2\phi_c P_n} + \frac{M}{\phi_b M_n} \leq 1.0 \\
\text{(H1-1b)}
\]

The available compressive strength \( \phi_c P_n \) is calculated using the equations of Chapter E and the available flexure strength \( \phi_b M_n \) is calculated using the equations of Chapter F. The axial force from factored loads \( P_u \) is calculated as before, but the bending moment from factored loads, \( M_u \), must include secondary moments which arise through the interaction of compressive forces and bending moments on a member. Procedures for calculating
Secondary moments are presented in Chapter C of the Manual, *Stability Analysis and Design*.

Secondary moments arise due to axial forces acting on member deflections. Secondary moments associated with member deflections between brace points are called P-δ effects (P-“little delta” effects), and secondary moments associated with joint displacements due to frame sidesway are called P-Δ effects (P-“big delta” effects).

![Figure 1. P-Δ versus P-δ secondary moments.](image)

Various procedures are presented by AISC for considering secondary moments. The Direct Analysis Method is presented in Chapter C and the alternate methods (the Effective Length Method and the First-Order Analysis Method) are presented in the appendix. Key features of the Direct Analysis Method include:

- A computer structural analysis program capable of performing a second-order analysis that considers both P-Δ and P-δ effects is required.
- The flexural stiffness, $E_l$, and the axial stiffness, $E_a$, of all members that contribute to the lateral stability of the structure are reduced to account for the effects of residual stresses.

\[
E_l^* = 0.8 \, t_b \, E_l \\
E_a^* = 0.8 \, E_a
\]

where: $t_b = 1.0$ if $P_u / P_y \leq 0.5$, else $t_b = 4 \left( \frac{P_u}{P_y} \right) \left[ 1 - \frac{P_u}{P_y} \right]$

Alternatively, $t_b$ can = 1.0 always if an additional notional load = 0.001 $Y_i$ to the notional load specified in the paragraph below.

- Notional (“imaginary) lateral loads equal to 0.002 times the gravity loads are added to account for “out of plumbness” of columns (top of column is not directly over the bottom) and curvature of columns (column is not completely straight).

\[
N_i = 0.002 \, Y_i, \quad Y_i = \text{factored gravity load for Level } i \text{ of frame}
\]
The origin of the 0.002 \( Y \) notional loads are demonstrated below.

\[
\sum M_{\text{base}} = 0
\]

\[
P \cdot \Delta = V \cdot L
\]

\[
V = \frac{P \Delta}{L} = \frac{P L}{500 L} = \frac{P}{500} = 0.002 P
\]

We will use the structural analysis program RISA3D to help us analyze steel frames using the Direct Analysis procedure. RISA3D can perform a P-\( \Delta \) analysis (based on nodal deflections) but not a P-\( \delta \) analysis (based on member deflections). We will add intermediate nodes (typically three) to columns to approximate a rigorous P-\( \delta \) analysis. RISA will reduce the flexural and axial stiffnesses of appropriate members, including iterative calculation of \( \tau_b \), to calculate secondary moments. Notional lateral loads equal to 0.002 times the total factored gravity load will need to be added for each load combination.