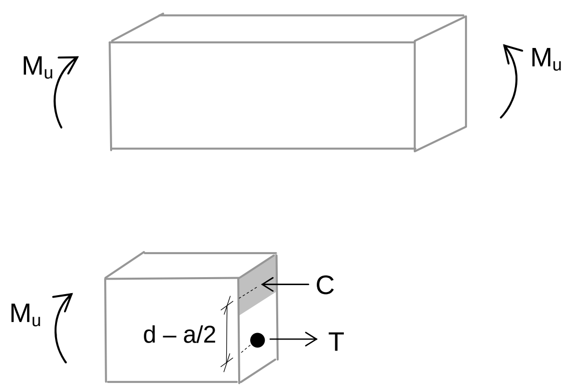
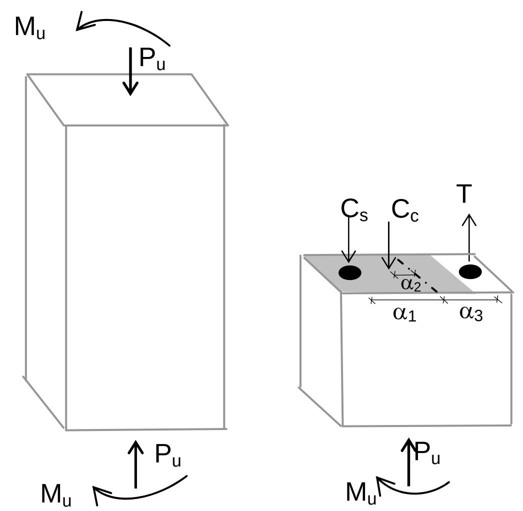
Analysis of Combined Axial and Bending Loads on Columns

Axial loads and bending moments both cause normal stresses on the column cross-section. We analyze the normal stresses from these combined loads in the same way that we analyze the normal stresses due to bending only in a beam, with two exceptions.

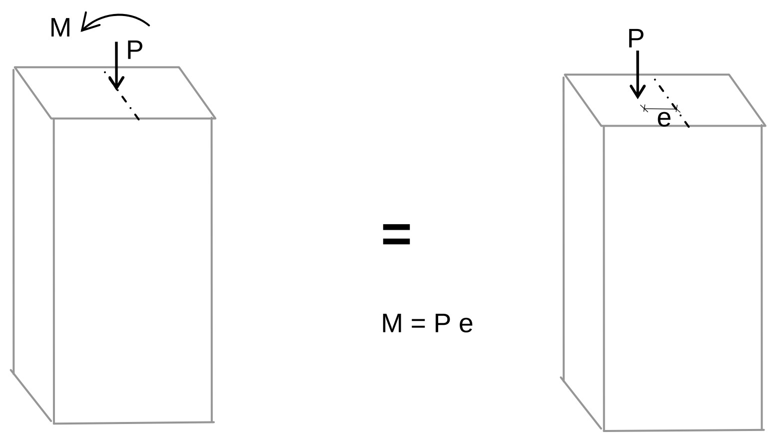
1. The sum of the normal stresses is now equal to the axial load (P_u), instead of equal to zero, and
2. We sum moments about the centroid of the column cross-section, instead of the centroid of the compressive stress on the concrete.

Beam	Column
 $\Sigma F = 0, \quad -C + T = 0$ $\Sigma M = M_u, \quad T \times (d - a/2) = M_u$	 $\Sigma F = P_u, \quad -C_s - C_c + T = P_u$ $\Sigma M = M_u, \quad C_s\,\alpha_1 + C_c\,\alpha_2 + T\,\alpha_3 = M_u$

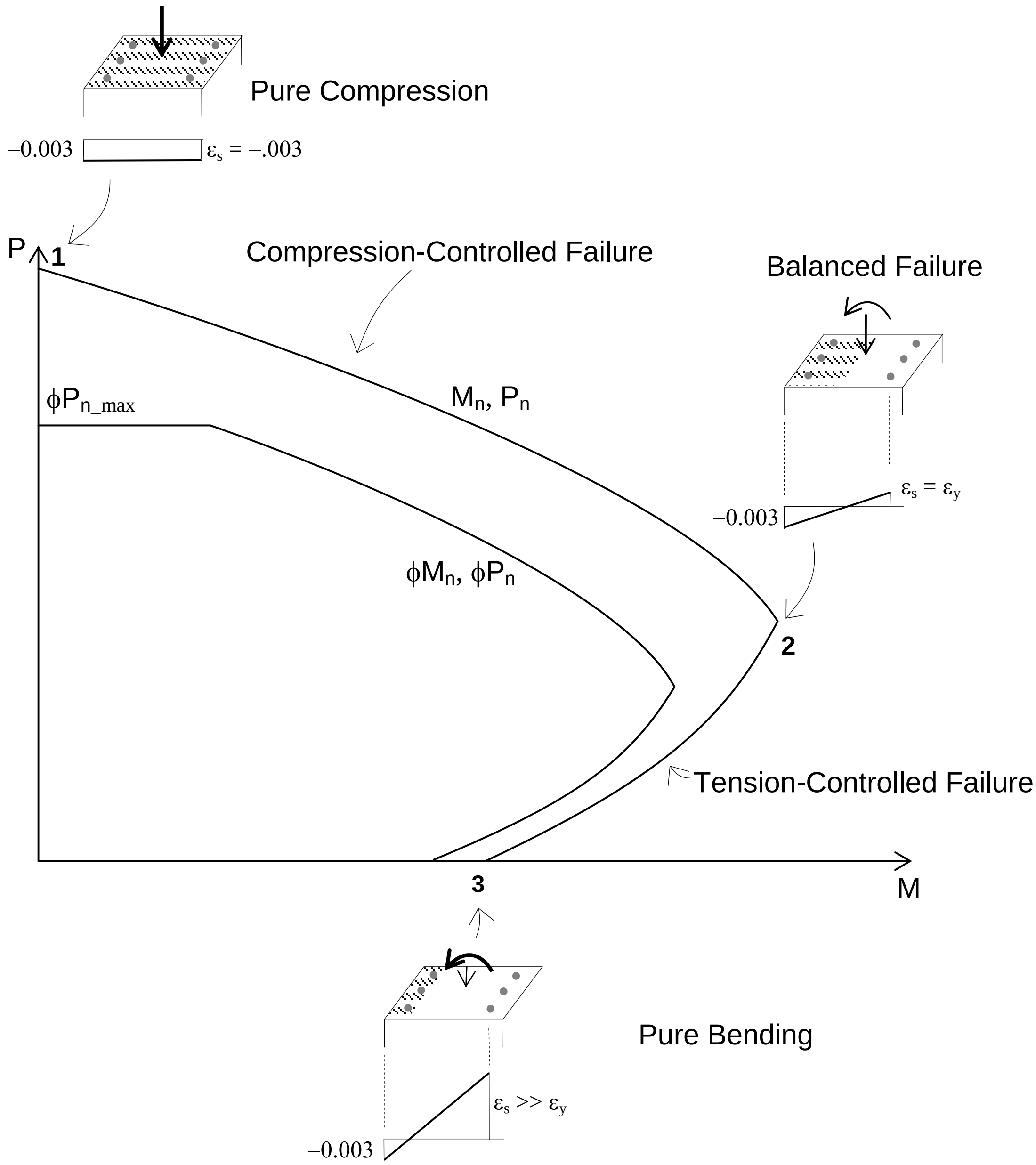
We calculate the loads on a column at ultimate strength just as we do for a beam:

1. Assume a strain profile for the column cross-section. Ultimate strength of a column occurs when the compressive strain in the concrete reaches 0.003, just as for a beam
2. Calculate the stresses in the concrete and steel.
3. Calculate the stress resultants.
4. The sum of the stress resultants is equal to the axial capacity of the column (P_n)
5. The sum of the moments caused by each stress resultant about the centroid of the column is equal to the moment capacity of the column (M_n).

Whereas a beam has only one moment capacity, a column has different axial and moment capacities for each ratio of M_n / P_n. This ratio is called the load eccentricity for the reason demonstrated in the figure below.



Column Interaction Diagram. The plot of axial capacity (P_n) vs. moment capacity (M_n) is called an interaction diagram. Each point on the interaction diagram is associated with a unique strain profile for the column cross-section. An interaction diagram has three key points, as shown in the figure below. Each point and each region between the points is discussed below.



Point 1: The column is in pure compression. The maximum axial capacity of the column occurs in this state.

Point 1 to Point 2 (*compression-controlled failure)*: The concrete crushes before the tension steel (layer furthest from the compression face) yields. Moment capacity decreases because the steel does not reach its full strength.

Point 2 (*Balanced failure)*: A so-called "balanced" failure occurs when the concrete crushes (ε_c = -0.003) at the same the tension steel yields (ε_s = 0.002).

Point 2 to Point 3 (*tension-controlled failure)*: As compression force is applied to the section, the compression area can increase beyond the area balanced by the tension steel. Larger compression force leads to larger moment.

Point 3: The column behaves as a beam. The compression area is limited by the area balanced by the tension steel.

<u>Strength Reduction Factor.</u> The reduced nominal axial capacity (ϕP_n) and the reduced nominal moment capacity (ϕM_n) are obtained by calculating the strength reduction factor (ϕ) based on the strain in the tension steel (the layer furthest from the compression face).

<u>Max. Axial Capacity.</u> ACI limits the axial force in a column (section 10.3.6, pg 123) to

$\phi P_{n,\max} = 0.85\phi[0.85 f_c^{'}(A_g - A_s) + f_y A_s]$ (flat portion at top of ϕM_n, ϕP_n curve)

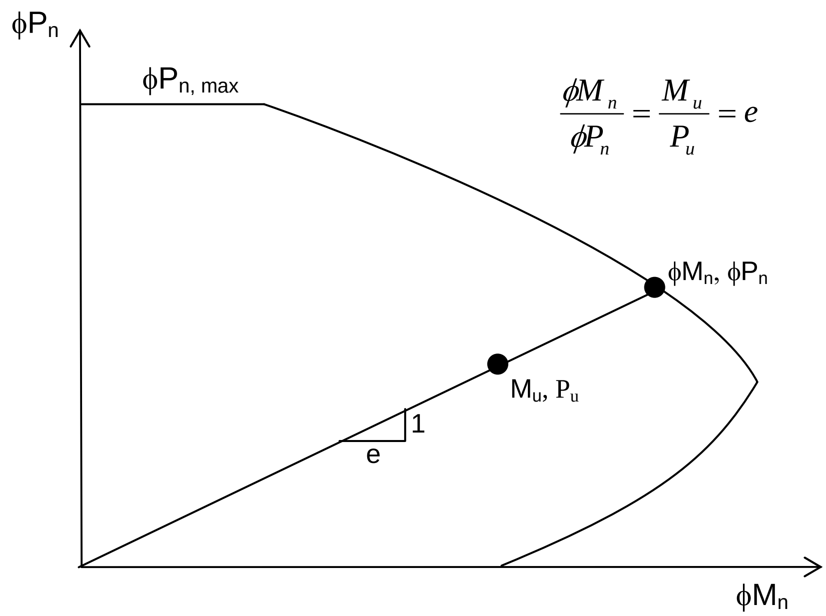
accounts for accidental eccentricity

Various methods exist for checking the combined normal stresses due to axial and bending in a column. Two methods are discussed here:

1) Single Point—useful when checking column for only one set of loads
2) Multi-point (full interaction diagram) —useful when checking column for multiple sets of loads

<u>Capacity Check for One Set of Loads</u>

Every point on the interaction diagram has a unique ratio of $\frac{\phi M_n}{\phi P_n} = e$. Therefore, if $\frac{\phi M_n}{\phi P_n} = \frac{M_u}{p_u} = e$ and $\phi M_n > M_u$ and $\phi P_n > P_u$, then the column is adequate.



Example

Check a 16" x 16" column with 5 #9 bars in each face to see if it is adequate for $P_u = 390^k$, $M_u = 220^{k\text{-}ft}$. f'c = 3000 psi, fy = 60,000 psi.

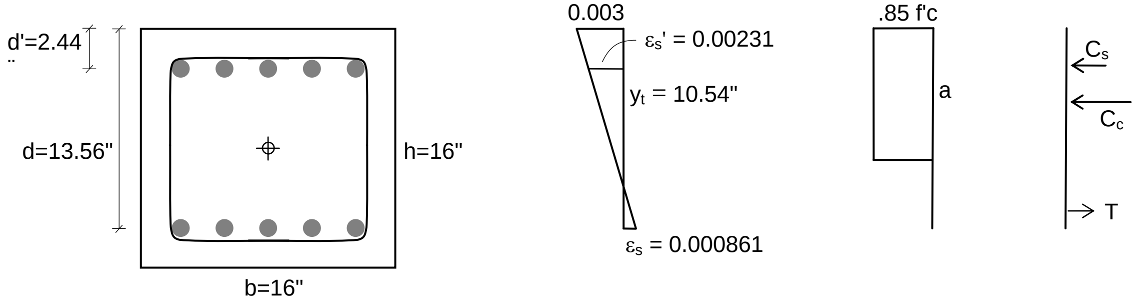
1. Compute eccentricity of loads:

$$\boxed{e = \frac{M_u}{P_u} = \frac{220^{k-ft}}{390^k} = 0.564^{ft}}$$

2. Use a spreadsheet (e.g. HW #3) to calculate the y_t value to give $\frac{\phi M_n}{\phi P_n} = e = 0.564$

   y_t = 10.54" for $\frac{\phi M_n}{\phi P_n} = e = 0.564^{ft}$

3. Construct the strain profile, calculate stresses in the concrete and each rebar layer, then calculate internal forces.



d' = 1.5" + 3/8" + 9/16" = 2.44" (assume #3 ties)

$C_c = 0.85\ f'c\ a\ b = 0.85(3^{ksi}).85(10.54")16" = 365^k$

$C_s = A_s'\ (f_y - .85f'c) = (5)1.00in^2\ (60^{ksi} - .85(3^{ksi})) = 287^k$ ($f_s' = f_y$ since $\varepsilon_s' > \varepsilon_y$)

$T = A_s\ f_s = 5.00in^2(0.000861)29{,}000^{ksi} = 125^k$

$P_n = \Sigma F = 365^k + 287^k - 125^k = 528^k$ (take compressive forces as +'ve)

$$M_n = \Sigma M = \left[365^k\left(\frac{16"}{2} - \frac{.85 \times 10.54"}{2}\right) + 287^k\left(\frac{16"}{2} - 2.44"\right) + 125^k\left(13.56" - \frac{16"}{2}\right)\right]\frac{1^{ft}}{12^{in}} = 298^{k-ft}$$

$\phi = 0.65$ since $\varepsilon_s < 0.002$

$$\phi P_n = 0.65(528^k) = 343^k$$

$$\phi M_n = 0.65(298^{k-ft}) = 194^{k-ft}$$

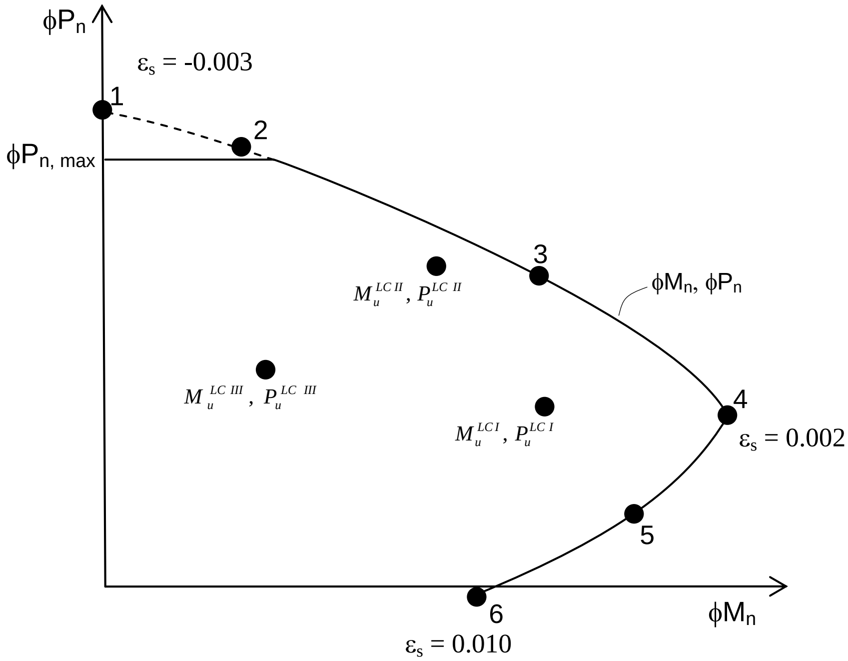
$$e = \frac{\phi M_n}{\phi P_n} = \frac{194^{k-ft}}{343^k} = 0.564^{ft} = \frac{M_u}{P_u},\quad y_t \text{ from spreadsheet OK}$$

But $\phi M_n = 194^{k\text{-}ft} < 220^{k\text{-}ft} = M_u$, NG

$\phi P_n = 343^k < 390^k = P_u$, NG

Capacity Check for Multiple Load Sets

The capacity of a column with several sets of loads (e.g. from different load combinations) can most easily be checked by generating a column interaction diagram.



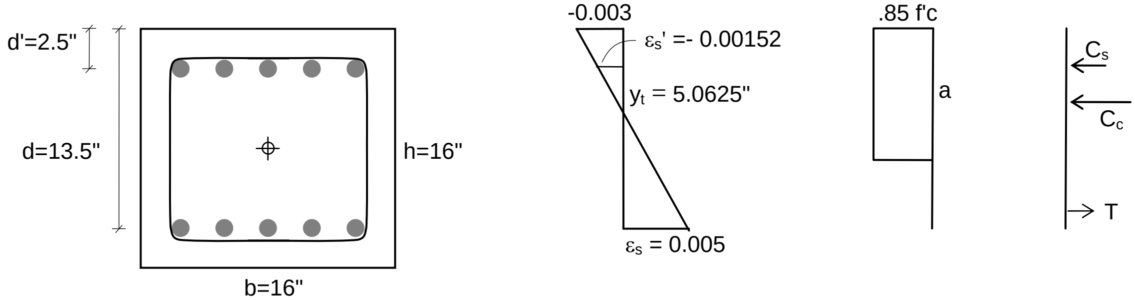
A point on the column interaction diagram can be calculated by assuming a strain profile in the column and calculating the resulting ϕM_n, ϕP_n. The strain profiles are known for Point 1 (ε_s = -0.003) and Point 4 ($\varepsilon_s = \varepsilon_y$). Point 6 can typically be calculated using ε_s = 5 ε_y = 0.01. Ideally, Point 2 should be just slightly greater than ϕ P_{n_max}, and Point 3 and Pont 5 midway between adjacent points.

Example: Pt. 5

Let $\varepsilon_s = 0.005$

f'c = 3 ksi, 5 #9 bars in each face

tension = +'ve



$$\frac{-0.003}{y_t} = \frac{-0.003-(+0.005)}{13.5}, \quad y_t = 5.0625"$$

$$\frac{\varepsilon_{s'}}{5.0625"-2.5"} = \frac{-0.003}{5.0625"}, \quad \varepsilon_{s'} = -0.00152$$

$a = b_1\, y_t = 0.85\ (5.0625") = 4.303"$

$C_c = -0.85\ f'c\ a\ b = -0.85(3^{ksi})\ 4.303"(16") = -176^k$

$f_{s'} = 29{,}000ksi\ (-0.00152) = -44.1^{ksi}$, $> -60^{ksi}$, OK

$C_s = A_s'\ [f_{s'} - (-.85f'c)] = (5)1.00in^2\ [-44.1^{ksi} + .85(3^{ksi})] = -208^k$

$T = A_s\ f_s = 5.00in^2(60{,}000^{ksi}) = 300^k$ since $\varepsilon_s > \varepsilon_y$

$P_n = \Sigma F = -208^k + -176^k + 300^k = -84^k$

$$M_n = \Sigma M = \left[-208^k\left(\frac{16"}{2}-2.5"\right)+(-176^k)\left(\frac{16"}{2}-\frac{.4.303"}{2}\right)+300^k\left(\frac{16"}{2}-13.5"\right)\right]\frac{1^{ft}}{12^{in}} = -319^{k-ft}$$

$\phi = 0.90$ since $\varepsilon_s = 0.005$

$$\phi P_n = 0.90(-84^k) = -76^k$$

$$\phi M_n = 0.90(-319^{k-ft}) = -287^{k-ft}$$