Column Design Example

Design a square column with bars in two faces for the following conditions.

\[ P_u = 600^k \]  \( f_c = 4000 \text{ psi} \)  Unsupported length = 12', part of braced frame

\[ M_u = 200^k-\text{ft} \]  \( f_y = 60,000 \text{ psi} \)  cover = 1.5"

1. Develop an equation for selecting the preliminary column dimensions. Use the equation for \( \phi P_{n,\text{max}} \) and assume that \( \frac{A_g}{A_s} = \rho_g = 0.015 \), from \( \frac{A_s}{A_g} = 0.01 \) to 0.04

and assume \( f_y = 60 \text{ ksi} \)

set \( P_u = \phi P_{n,\text{max}} = 0.80 \phi [0.85 f_c'(A_g - A_s) + f_y A_s] \)  ACI Eqn 10-2

\[ P_u = (0.80)(0.65)(0.85) f_c'(A_g - A_s) + (0.80)(0.65) f_y A_s \rho_g \]

\[ P_u = 0.442 f_c' A_g (1 - \rho_g) + 0.520 f_y A_g \rho_g = 0.442 (1 - 0.015) f_c' A_g + 0.520 (60 \text{ ksi})(0.015) A_g \]

\[ P_u = A_g [0.435 f_c' + 0.468] \approx A_g [0.40 f_c' + 0.45], \text{ or} \]

\[ A_g \approx \frac{P_u}{0.40 f_c' + 0.45} = \frac{600^k}{0.40 (4 \text{ ksi}) + 0.45} = 293.7 \text{ in}^2 = 17.1 \text{ in} \times 17.1 \text{ in} \]

Try an 18" x 18" column.

2. Check if the column is slender.

ACI 10.10.1: slenderness can be ignored if \( \frac{k l_u}{r} \leq 34 - 12 \frac{M_1}{M_2} \) and the column is part of a braced frame. In a braced frame, some other element besides the columns resist sidesway, such as the shear walls in an elevator shaft.

Where:

- \( k \) = effective length factor (assume equal to 1 for this example)
- \( l_u \) = unbraced length = 12'
- \( r \) = radius of gyration = 0.3 h (ACI 10.10.1.2)
- \( r = 0.3 \text{ (18'')} = 5.40'' \)
- \( M_2 \) is larger of two moments

Assume (for this example)

\[ \frac{M_1}{M_2} = +0.5 \text{(conservative)} \]

double curvature

\[ \frac{M_1}{M_2} = -\text{ve} \]

single curvature

\[ \frac{M_1}{M_2} = +\text{ve} \]
3. Select longitudinal reinforcement using trial and error and a spreadsheet that calculates the interaction diagram given the concrete dimensions and reinforcement layout. Using such a spreadsheet, you get the following results for 4 #9 bars in each face:

<table>
<thead>
<tr>
<th>Pt. #</th>
<th>$\varepsilon_t$ / $\varepsilon_y$</th>
<th>$\varepsilon_t$</th>
<th>$\phi P_n$</th>
<th>$\phi M_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.00299</td>
<td>-808</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.001</td>
<td>-808</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.0005</td>
<td>-807</td>
<td>121</td>
<td></td>
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<tr>
<td>4</td>
<td>0</td>
<td>-673</td>
<td>185</td>
<td></td>
</tr>
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<td>5</td>
<td>0.5</td>
<td>0.00103</td>
<td>-460</td>
<td>256</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.00207</td>
<td>-305</td>
<td>300</td>
</tr>
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</tr>
<tr>
<td>10</td>
<td>8</td>
<td>0.01655</td>
<td>111</td>
<td>189</td>
</tr>
</tbody>
</table>

4. Check the reinforcement ratio against ACI min and max \((ACI \: 10.9.1)\)

$$\rho_{\text{min}} = 0.01, \quad \rho_{\text{practical max}} = 0.04, \quad \rho_{\text{max}} = 0.08,$$

$$\rho = \frac{A_{st}}{A_g} = \frac{(8)(1.00\text{in}^2)}{(18\text{in})(18\text{in})} = 0.25, \quad \text{OK}$$

5. Select ties and tie spacing. \((ACI \: 7.10.5.1)\): min. tie size = #3 if longitudinal bars <= #10, else min. tie size = #4 bars.

therefore, use #3 bars for ties

max. tie spacing = min[ 16 $\phi_{\text{bar}}$, 48 $\phi_{\text{ties}}$, min[ b, h ]], \((ACI \: 7.10.5.2)\)
tie spacing = min[ 16 (1.128”), 48 (0.375”), min [18”, 18”] ]
tie spacing = min[ 18.0”, 18.0”, 18.0” ],

therefore, use tie spacing = 18”
6. Check if sufficient space between bars for concrete to pass when being placed in form. Critical location is at lap splices.

![Cross-section of Column](image)

*Min. bend diameter = 4 $\phi_{tie}$ for #5 and smaller ties (*ACI 7.2.2*), therefore minimum radius = 2 $\phi_{tie}$.

** Min. clear space between adjacent bars = \text{max}[1.5 \, \phi_{bar}, \ 1.5'']$, *ACI 7.6.3*.

\[
b_{\min} = 2 \left[ 1.5'' + 3 (0.375'') + 0.707 (1.128'') \right] + 3 \left[ 1.128'' + 1.692'' \right] \\
= \text{max}[1.5 (1.128'') + 1.5''] = 1.692''
\]

\[
b_{\min} = 2 \left[ 1.5'' + 3 (0.375'') + 0.707 (1.128'') \right] + 3 \left[ 1.128'' + 1.692'' \right] \\
= 15.3'' < 18'' = b, \text{ OK}
\]

7. Determine the number of ties required in each direction.

*ACI 7.10.5.3*: Ties should be arranged such that every corner longitudinal bar, and every alternate longitudinal bar shall have lateral support provided by the corner of a tie. Also, no longitudinal bar should be further than 6’’ clear from a longitudinal bar with lateral support.

Clear space between bars = \{18” – 2 \left[ 1.5'' + 3 (0.375'') \right] \} / 3 spaces - 1.128”

Clear space between bars = 3.12” < 6”, therefore only need ties for every alternate longitudinal bar.
8. Determine the splice length.

\textit{ACI 12.17.2.4} says that the required splice length can be multiplied by 0.83 if the total area of the ties > 0.0015 \( h_{tie} \) in both directions.

\[ 0.0015 \, h_{tie} = 0.0015 \times (18") \times (18") = 0.486 \text{ in}^2 \]

parallel to \( h \): \( A_{ties} = 3 \times (0.11 \text{ in}^2) = 0.33 \text{ in}^2 \)

parallel to \( b \): \( A_{ties} = 2 \times (0.11 \text{ in}^2) = 0.22 \text{ in}^2 \)

Since \( A_{ties} \) is not > 0.0015 \( h_{tie} \) in either direction (both directions would need to be satisfied), the reduction factor does not apply.

\textit{ACI 12.15.1}: Since all longitudinal bars will be spliced at the same location, this splice is a Class B splice and \( l_{splice} = 1.3 \, l_d \), where \( l_d \) is the development length.

\[
l_d = \frac{f_y \, \phi_{bar}}{[20 \text{ if bar size } > \#6] \sqrt{f'_c}} = \frac{60,000 \text{ psi} \, (1.128\text{")}}{20 \sqrt{4000 \text{ psi}}} = 53.5\text{”}
\]

therefore splice length = 1.3 \( (53.5\text{”}) = 69.6\text{”} \), use a splice length = 70”

Note: the splice length could be halved by using \#6 bars, but this would require a larger column and more ties.