Write out a complete step-by-step solution for the HW #2 problem (which was to write an equation for the vertical distance between the center of the circle and the x-axis in terms of “a”). Sketches are provided for you to illustrate the step. Each step must be written in complete sentences. Identify what you think are the trickiest steps with an asterisk.

1. Draw a vertical line from the center of the circle down to the x-axis and label the length of the line “y”. Label the distance from point “A” to the bottom of the line “a/2”.

2. Draw a line from point “A” to the center of the circle. Note that we could calculate y using the Pythagorean Theorem if we knew the length of “c”.

\[ c^2 = (a/2)^2 + y^2 \]

3. Extend line c up and to the right until it intersects arc CB. Label the length of this line as “a”, the radius of arc CB.

4. Note that if we knew the length of the extended portion of the line we could calculate c. This extended portion is equal to the radius of the circle. Label the extended portion of the line as “r”.

\[ c + r = a \]

* 5. We need to relate r in another equation besides the one we just wrote above. After a little trial-and-error, we find that we can draw a line from the center of the big circle to the center of one of the little half-circles, and the length of this line is the radius of the big circle plus the radius of the little half-circle! Label these lengths.

The radius of the half-circle is half of a/2 or a/4.

6. We now have another triangle. We can relate the lengths of the sides using the Pythagorean Theorem.

\[ (r + a/4)^2 = (a/4)^2 + y^2 \]
7. Let’s take stock of where we are. We have three equations and three unknowns (c, y and r). The equations appear to be independent (one equation is not a multiple of another equation (see pg. 291 in Schaum’s)). We want to solve these equations so that we can write y as a function of “a” only.

(1) \( c^2 = (a/2)^2 + y^2 \)

(2) \( c + r = a \)

(3) \( (r + a/4)^2 = (a/4)^2 + y^2 \)

8. We can solve Eqn (2) for “c” and substitute it into Eqn (1):

\( (2): c = a – r \)

\( (1): (a – r)^2 = (a/2)^2 + y^2 \)

9. We now have two equations with two unknowns (r and y). We can eliminate y by subtracting Eqn (3) from Eqn (1) to form Eqn (4):

\[ (1): (a – r)^2 = (a/2)^2 + y^2 \]
\[ (3): (r + a/4)^2 = (a/4)^2 + y^2 \]

\[ a^2 – 2ar + r^2 = a^2/4 + y^2 \]
\[ r^2 + 2ar/4 + a^2/16 = a^2/16 + y^2 \]

\[ a^2 – 2ar + r^2 – r^2 – 2ar/4 – a^2/16 = a^2/4 + y^2 – a^2/16 – y^2 \]
\[ a^2 (3/4) = a r (5/2) \]
\[ r = a (3/4)(2/5) = a (6/20) \]
\[ r = a (3/10) \]

10. Substitute this last eqn. into Eqn (3):

\[ 9/100a^2 + a(a(3/10))/2 = y^2 \]
\[ a^2(9/100 + 3/20) = a^2(9 + 15)/100 = a^2(24/100) = a^2(6/25) = y^2 \]

\[ y = \frac{\sqrt{6}}{5} a \]