Stiffness of Walls with Openings

Lateral force (due to wind or seismic load) is transmitted by roof or floor diaphragm to end walls.

Shear at top of wall is distributed to piers.

Blocks of rigid diaphragms at walls of multiple piers are indeterminate systems. Lateral force is distributed according to relative stiffness.

Stiffness of walls w/ openings

Review: parallel vs. series stiffness

Hooke's Law

\[ P = \frac{KL}{\Delta} \]

\[ P = K\Delta \]
Stiffnesses in parallel:

\[ F_1 = k_1 \Delta_1 \]
\[ F_2 = k_2 \Delta_2 \]
\[ \Delta = \Delta_1 = \Delta_2 \]
\[ F_1 = k_1 \Delta \]
\[ F_2 = k_2 \Delta \]

\[ F_1 + F_2 = F \]
\[ k_1 \Delta + k_2 \Delta = F \]
\[ (k_1 + k_2) \Delta = F \]
\[ k_{\text{total}} = k_1 + k_2 \]

Stiffnesses in series:

\[ F_1 = k_1 \Delta \]
\[ F_2 = k_2 \Delta \]

\[ F_1 + F_2 = F \]
\[ \frac{F_1}{k_1} + \frac{F_2}{k_2} = \Delta = F \]
\[ F (\frac{1}{k_1} + \frac{1}{k_2}) = \Delta \]
\[ k_{\text{total}} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} \]
Wall with opening:

By superposition:

\[
\frac{1}{K_{\text{Tot}}} = \frac{1}{K_{\text{solid}}} + \frac{1}{K_{\text{piers}}} \\
= \frac{1}{K_{\text{solid}}} - \frac{1}{K_{\text{piers}}}
\]

\[
\frac{1}{K_{\text{Tot}}} = \frac{1}{K_{\text{solid}}} - \frac{1}{K_{\text{piers}}}
\]

and

\[
K_{\text{piers}} = K_{\text{pier1}} + K_{\text{pier2}}
\]

Stiffnesses in parallel:

\[
\text{same } \Delta \\
\text{different } F
\]
Pier Stiffness:

\[ P_i = \frac{\Delta P}{\Delta L} = \frac{P}{L} \]

\[ \Delta = \frac{P_i}{A} \]

\[ \Delta_{\text{bending}} + \Delta_{\text{shear}} \]

\[ \kappa_{\text{bending}} = \frac{P}{\Delta} = \frac{3EI}{h^3} \quad \text{for cantilever walls} \]

\[ = \frac{12EI}{h^3} \quad \text{for walls with rotations restrained at top} \]

\[ \kappa_{\text{shear}} = \frac{3A}{1.2h} \]

\[ \frac{1}{\kappa_{\text{TOT}}} = \frac{1}{\kappa_{\text{bending}}} + \frac{1}{\kappa_{\text{shear}}} \]

\[ \frac{1}{\kappa_{\text{TOT}}} = \frac{h^3}{3EI} + \frac{1.2h}{GA} \quad \text{for cantilever walls} \]

\[ \begin{align*}
I &= \frac{1}{12} \ell^2 \\
G &= \frac{E}{2(1+\nu)} \quad \nu = \text{Poisson's ratio} = 0.3 \\
G &= \frac{E}{2(1+\nu)} = 0.4E \\
A &= \ell L
\end{align*} \]
\[ \frac{1}{k_{\text{tot}}} = \frac{h^3}{3 \varepsilon \mu^2} + \frac{1.2 \varepsilon h}{\mu^2} \]

\[ \frac{1}{k_{\text{tot}}} = \frac{1}{\varepsilon \mu} \left( 4 \left( \frac{h}{2} \right)^3 + 3 \frac{h}{2} \right) \quad \text{for cantilever walls} \]

\[ \frac{1}{k_{\text{tot}}} = \frac{1}{\varepsilon \mu} \left( \left( \frac{h}{2} \right)^3 + 3 \frac{h}{2} \right) \quad \text{for rotationally restrained at top} \]

**Example**

\[ \text{Elev. - East Wall} \]

\[ \text{WL} = 20 \text{ psf} \]

\[ V_w \text{ at top of east wall due to wind blowing on south wall (MSD, pg. 11-3, 11-4)} \]

\[ \Sigma M \text{ base of South wall} = 0 \]

\[ (\text{WL})(19')(\frac{19'}{2}) = (R_{\text{diaph}})(16') \quad \text{WL} \]

\[ R_{\text{diaph}} = 226 \text{ psf} \]

Section of South Wall
WL from roof diaphragm to east wall = \( \frac{V_{\text{wall}}}{V_{\text{east}}/4} \)

\[ V_{\text{east}} = (226 \times f) \left( \frac{264.8}{4} \right) = 11,600 \text{ lb} \]

Use tributary area, be diaphragm is flexible.

Distribution of shear within east wall

Wall is perforated
Wall is reinforced (no control joints)

\[ K_{\text{wall}} = \frac{1}{K_{\text{wall, solid}}} - \frac{1}{K_{\text{A, open}}} + \frac{1}{K_{\text{A}}} \]

\[ K_{\text{A}} = K_{B} + K_{C} \]

\[ \frac{1}{K_{B}} = \frac{1}{K_{B, solid}} - \frac{1}{K_{D, solid}} + \frac{1}{K_{1,2}} \]

\[ K_{1,2} = K_{1} + K_{2} \]
\[
\frac{1}{K_i} = \frac{1}{10 \frac{\text{in}}{\text{in}}} \left( \left( \frac{n'}{30} \right)^3 + 3 \left( \frac{n'}{30} \right) \right) = 0.0713 \frac{\text{in}}{\text{lb}}
\]

\[
K_i = \frac{1}{10} = 14.03 \frac{\text{lb}}{\text{in}}
\]

\[
\frac{1}{K_2} = \frac{1}{10 \frac{\text{in}}{\text{in}}} \left( \frac{n'}{2.8} \right)^3 + 3 \left( \frac{n'}{2.8} \right) = 2.596 \frac{\text{lb}}{\text{in}}
\]

\[
K_2 = 0.39 \frac{\text{lb}}{\text{in}}
\]

\[
\frac{1}{K_8} = 0.0855 \frac{\text{in}}{\text{lb}} - 0.0591 \frac{\text{in}}{\text{lb}} + \frac{14.42 \frac{\text{lb}}{\text{in}}}{10}
\]

\[
\frac{1}{K_8} = 0.0957 \frac{\text{in}}{\text{lb}}
\]
\[ K_A = \frac{1}{0.0957} + \frac{1}{0.0855} = -22.15 \text{ in} \]

\[ \frac{1}{K_{wall}} = 0.0625 \text{ in} - 0.0303 \text{ in} + \frac{1}{22.15} \text{ in} \]

\[ \frac{1}{K_{wall}} = 0.0693 \text{ in} \]

\[ K_{wall} = 14.4 \text{ in} \]

Use \( K_{wall} \) to calc. shear to eastwall if roof is a rigid diaphragm.

Use \( K_A \) to divide wall shear to left & right side of opening.

Use \( K_A \) to divide shear on left side of opening to piers 1 & 2.

\[ V_{pier 3} = V_{area C} = \frac{K_A}{K_A + K_C} \]

\[ V_{pier 3} = (11.6600) \left( \frac{0.855}{0.855 + 0.855} \right) \]

\[ V_{pier 3} = 6,130 \text{ lb} \]

\[ V_{area B} = 11,600 \left( \frac{1.0957}{1.0957 + 1.0957} \right) = 5,470 \text{ lb} \]

\[ V_{pier 1} = 5,470 \left( \frac{14.03}{14.03 + 0.39} \right) \]

\[ V_{pier 1} = 5,320 \text{ lb} \]

\[ V_{pier 2} = 5,470 \left( \frac{0.39}{14.03 + 0.39} \right) \]

\[ V_{pier 2} = 150 \text{ lb} \]