SEISMIC PROTECTIVE SYSTEMS:
PASSIVE ENERGY DISSIPATION

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Major Objectives

• Illustrate why use of passive energy dissipation systems may be beneficial
• Provide overview of types of energy dissipation systems available
• Describe behavior, modeling, and analysis of structures with energy dissipation systems
• Review developing building code requirements
Outline: Part I

• Objectives of Advanced Technology Systems and Effects on Seismic Response
• Distinction Between Natural and Added Damping
• Energy Distribution and Damage Reduction
• Classification of Passive Energy Dissipation Systems
Outline: Part II

• Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
• Models for Velocity-Dependent Dampers
• Effects of Linkage Flexibility
• Displacement-Dependent Damping Systems: Steel Plate Dampers, Unbonded Brace Dampers, and Friction Dampers
• Concept of Equivalent Viscous Damping
• Modeling Considerations for Structures with Passive Energy Dissipation Systems
Outline: Part III

- Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
- Representations of Damping
- Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
- Summary of MDOF Analysis Procedures
Outline: Part IV

- MDOF Solution Using Complex Modal Analysis
- Example: Damped Mode Shapes and Frequencies
- An Unexpected Effect of Passive Damping
- Modeling Dampers in Computer Software
- Guidelines and Code-Related Documents for Passive Energy Dissipation Systems
Outline: Part I

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Objectives of Energy Dissipation and Seismic Isolation Systems

• Enhance performance of structures at all hazard levels by:
  ▪ Minimizing interruption of use of facility (e.g., Immediate Occupancy Performance Level)
  ▪ Reducing damaging deformations in structural and nonstructural components
  ▪ Reducing acceleration response to minimize contents-related damage
Effect of Added Damping (Viscous Damper)

- Decreased Displacement
- Decreased Shear Force
Effect of Added Stiffness (Added Bracing)

- Decreased Displacement
- Increased Shear Force
Effect of Added Damping and Stiffness (ADAS System)

- Decreased Displacement
- Decreased Shear (possibly)
Effect of Reduced Stiffness (Seismic Isolation)

- Decreased Shear Force
- Increased Displacement

Pseudo-Spectral Acceleration, g vs. Spectral Displacement, Inches

- 5% Damping
- 10%
- 20%
- 30%
- 40%

T = 0.50, 1.0, 1.5, 2.0, 3.0, 4.0
Effect of Reduced Stiffness (Seismic Isolation with Dampers)

- Decreased Shear
- Increased Displacement

Pseudo-Spectral Acceleration, g vs. Spectral Displacement, Inches

5% Damping
10%
20%
30%
40%
Effect of Damping and Yield Strength on Deformation Demand

![Graph showing the effect of damping and yield strength on peak displacement. The x-axis represents the damping ratio (%) and the y-axis represents the peak displacement (in). The graph includes lines for different yield strengths: 10 kips (red diamonds), 20 kips (green squares), and 30 kips (blue triangles).}
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Distinction Between Natural and Added Damping

Natural (Inherent) Damping

\( \xi \) is a structural property, dependent on system mass, stiffness, and inherent energy dissipation mechanisms

\[ \xi_{\text{NATURAL}} = 0.5 \text{ to } 7.0\% \]

Added Damping

\( \xi \) is a structural property, dependent on system mass, stiffness, and the added damping coefficient C

\[ \xi_{\text{ADDED}} = 10 \text{ to } 30\% \]
Response of Bare Frame Before and After Adding Ballast

Model Weight
Bare Model 18 kips
Loaded Model 105 kips
Change in Damping and Frequency with Accumulated Damage

PEAK GROUND ACCELERATION, %G

FREQUENCY, HZ
DAMPING, % CRITICAL
Outline: Part I

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Reduction in Seismic Damage

Energy Balance:

\[ E_I = E_S + E_K + (E_{DI} + E_{DA}) + E_H \]

Damage Index:

\[ DI(t) = \frac{u_{max}}{u_{ult}} + \rho \frac{E_H(t)}{F_y u_{ult}} \]

Source: Park and Ang (1985)
Duration-Dependent Damage Index

\[ DI(t) = \frac{u_{\text{max}}}{u_{\text{ult}}} + \rho \frac{E_{H}(t)}{F_{y}u_{\text{ult}}} \]

Source: Park and Ang (1985)

- \( u_{\text{max}} \) = maximum displacement
- \( u_{\text{ult}} \) = monotonic ultimate displacement
- \( \rho \) = calibration factor
- \( E_{H} \) = hysteretic energy dissipated
- \( F_{y} \) = monotonic yield force

Diagram:
- \( DI \) vs. Damage State
- \( DI \) vs. Collapse State
- \( DI \) range from 0.0 to 1.0
Damping Reduces Hysteretic Energy Dissipation Demand
Effect of Damping and Yield Strength on Hysteretic Energy

![Graph showing the effect of damping and yield strength on hysteretic energy.](image_url)
Energy and Damage Histories, 5% Damping

\[ E_I = 260 \]

\[ E_{DI} + E_{DA} \]

\[ E_H \]

Max = 0.55

Analysis performed on NONLIN
Energy and Damage Histories, 20% Damping

\[ E_I = 210 \]

\[ E_{DI} + E_{DA} \]

\[ E_H \]

Max = 0.30
Reduction in Damage with Increased Damping

![Graph showing reduction in damage with increased damping](image-url)
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Classification of Passive Energy Dissipation Systems

Velocity-Dependent Systems
- Viscous fluid or viscoelastic solid dampers
- May or may not add stiffness to structure

Displacement-Dependent Systems
- Metallic yielding or friction dampers
- Always adds stiffness to structure

Other
- Re-centering devices (shape-memory alloys, etc.)
- Vibration absorbers (tuned mass dampers)
Outline: Part II

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- Modeling Considerations for Structures with Passive Damping Systems
Possible Damper Placement Within Structure
Chevron Brace and Viscous Damper
Diagonally Braced Damping System
Fluid Dampers within Inverted Chevron Brace

Pacific Bell North Area Operation Center (911 Emergency Center)
Sacramento, California
(3-Story Steel-Framed Building Constructed in 1995)

62 Dampers: 30 Kip Capacity, +/-2 in. Stroke
Fluid Damper within Diagonal Brace

San Francisco State Office Building
San Francisco, CA

Huntington Tower
Boston, MA
Toggle Brace Damping System

\[ AF = \frac{U_D}{U_1} = \frac{\sin \theta_2}{\cos(\theta_1 + \theta_2)} \]
Toggle Brace Deployment

Huntington Tower, Boston, MA
- New 38-story steel-framed building
- 100 direct-acting and toggle-brace dampers
- 1300 kN (292 kips), +/- 101 mm (+/- 4 in.)
- Dampers suppress wind-induced vibration
Harmonic Behavior of Fluid Damper

\[ u(t) = u_0 \sin(\bar{\omega}t) \]

\[ P(t) = P_0 \sin(\bar{\omega}t)\cos(\delta) + P_0 \cos(\bar{\omega}t)\sin(\delta) \]

**Note:** Damping force 90° out-of-phase with elastic force.
\[ P(t) = K_S u(t) + C \dot{u}(t) \]

- \( K_S = \frac{P_0}{u_0} \cos(\delta) \) \hspace{1cm} \text{Storage Stiffness}
- \( K_L = \frac{P_0}{u_0} \sin(\delta) \) \hspace{1cm} \text{Loss Stiffness}
- \( C = \frac{K_L}{\bar{\omega}} \) \hspace{1cm} \text{Damping Coeff.}
- \( \delta = \sin^{-1}\left(\frac{P_Z}{P_0}\right) \) \hspace{1cm} \text{Phase Angle}

\[ P_Z = K_L u_o = P_o \sin(\delta) \] \hspace{1cm} \text{Damper Displacement, } u

\[ E_D = \pi P_Z u_o = \pi P_o u_o \sin(\delta) \]
**Frequency-Domain Force-Displacement Relation**

\[ P(t) = K_S u(t) + C \dot{u}(t) \]

Apply Fourier Transform:

\[ P(\overline{\omega}) = K_S u(\overline{\omega}) + K_L i \overline{\omega} u(\overline{\omega})/\overline{\omega} \]

\[ P(\overline{\omega}) = [K_S + iK_L] u(\overline{\omega}) \]

Complex Stiffness:

\[ K^*(\overline{\omega}) = \frac{P(\overline{\omega})}{u(\overline{\omega})} \]

\[ P(\overline{\omega}) = K^*(\overline{\omega}) u(\overline{\omega}) \]

Note: \[ \Re(K^*) = K_S \quad \text{and} \quad \Im(K^*) = K_L \]
Dependence of Storage Stiffness on Frequency for Typical “Single-Ended” Fluid Damper

Storage Stiffness (lb/in)

Excitation Frequency (Hz)

Cutoff Frequency
Dependence of Damping Coefficient on Frequency for Typical "Single-Ended" Fluid Damper
Dependence of Phase Angle on Frequency for Typical “Single-Ended” Fluid Damper

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Phase Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td>120</td>
</tr>
<tr>
<td>25</td>
<td>120</td>
</tr>
</tbody>
</table>
Dependence of Damping Coefficient on Temperature for Typical Fluid Damper
Behavior of Fluid Damper with Zero Storage Stiffness

\[ K_S = 0 \implies \delta = 90^\circ \]

\[ P(t) = C\dot{u} = \frac{KL}{\bar{\omega}}\ddot{u} \]

\[ E_d = \pi P_o u_o \]
Actual Hysteretic Behavior of Fluid Damper

Harmonic Loading

Seismic Loading

Source: Constantinou and Symans (1992)
Force-Velocity Behavior of Viscous Fluid Damper

\[ P(t) = C|\dot{u}|^\alpha \text{sgn}(\dot{u}) \]
Nonlinear Fluid Dampers

\[ P(t) = C \left| \dot{u} \right|^\alpha \text{sgn} (\dot{u}) \]

\( \alpha = 0.1 \)

\( \alpha = 1.0 \)
Energy Dissipated Per Cycle for Linear and Nonlinear Viscous Fluid Dampers

Linear Damper: \[ E_D = \pi P_o u_o \]

Nonlinear Damper: \[ E_D = \lambda P_o u_o \]

\[ \lambda = 4 \times 2^{\alpha} \frac{\Gamma^2 \left(1 + \frac{\alpha}{2}\right)}{\Gamma(2 + \alpha)} \]

\[ \Gamma = \text{Gamma Function} \]
Relationship Between $\lambda$ and $\alpha$ for Viscous Fluid Damper

![Graph showing the relationship between $\lambda$ and $\alpha$ for a viscous fluid damper. The graph includes points with AF values of 1.24, 1.11, and 1.0. The equation $AF = \lambda/\pi$ is also shown.]
Relationship Between Nonlinear and Linear Damping Coefficient for Equal Energy Dissipation Per Cycle

\[ \frac{C_{NL}}{C_L} = \frac{\pi}{\lambda} \left( \frac{u_o \omega}{\lambda} \right)^{1-\alpha} \]

Note: Ratio is frequency- and displacement-dependent and is therefore meaningful only for steady-state harmonic response.
Ratio of Nonlinear Damping Coefficient to Linear Damping Coefficient (For a Given Loading Frequency)

Loading Frequency = 1 Hz (6.28 rad/sec)
Ratio of Nonlinear Damping Constant to Linear Damping Constant (For a Given Maximum Displacement)

Maximum Displacement = 1

![Graph showing the ratio of nonlinear damping constant to linear damping constant for different load frequencies.]

- Load Freq = 1/3 Hz (2.09 rad/s)
- Load Freq = 1/2 Hz (3.14 rad/s)
- Load Freq = 2 Hz (6.28 rad/s)
Example of Linear vs Nonlinear Damping

Frequency = 1 Hz (6.28 rad/s)
Max. Disp. = 10.0 in.

\( \lambda = 3.58 \)

\( C_{\text{Linear}} = 10.0 \text{ k-sec/in} \)
\( C_{\text{Nonlinear}} = 69.5 \text{ k-sec}^{0.5/\text{in}}^{0.5} \)
Recommendations Related to Nonlinear Viscous Dampers

• Do NOT attempt to linearize the problem when nonlinear viscous dampers are used. Perform the analysis with discrete nonlinear viscous dampers.

• Do NOT attempt to calculate effective damping in terms of a damping ratio ($\xi$) when using nonlinear viscous dampers.

• DO NOT attempt to use a free vibration analysis to determine equivalent viscous damping when nonlinear viscous dampers are used.
Advantages of Fluid Dampers

• High reliability
• High force and displacement capacity
• Force Limited when velocity exponent < 1.0
• Available through several manufacturers
• No added stiffness at lower frequencies
• Damping force (possibly) out of phase with structure elastic forces
• Moderate temperature dependency
• *May* be able to use linear analysis
Disadvantages of Fluid Dampers

• Somewhat higher cost
• Not force limited (particularly when exponent = 1.0)
• Necessity for nonlinear analysis in most practical cases (as it has been shown that it is generally not possible to add enough damping to eliminate all inelastic response)
Viscoelastic Dampers

- Developed in the 1960’s for Wind Applications

Section A-A
Implementation of Viscoelastic Dampers

Building 116, US Naval Supply Facility, San Diego, CA
- Seismic Retrofit of 3-Story Nonductile RC Building
- 64 Dampers Within Chevron Bracing Installed in 1996
Harmonic Behavior of Viscoelastic Damper

\[ u(t) = u_0 \sin(\bar{\omega} t) \]

\[ P(t) = P_0 \sin(\bar{\omega} t) \cos(\delta) + P_0 \cos(\bar{\omega} t) \sin(\delta) \]
$P(t) = K_S u(t) + C \dot{u}(t)$

$K_S = \frac{G' A}{h}$

$K_L = \frac{G'' A}{h}$

$C = \frac{K_L}{\omega}$

$\delta = \sin^{-1} \left( \frac{\tau_Z}{\tau_0} \right)$

$\eta = \frac{G''(\omega)}{G'(\omega)} = \tan(\delta)$

$G' = \text{Storage Modulus}$

$G'' = \text{Loss Modulus}$

$\gamma_0$

$\tau_Z = G'' \gamma_o = \tau_0 \sin(\delta)$

$\tau(t) = G' \gamma(t) + G'' \dot{\gamma}(t) / \omega$

$E_D = \pi \tau_Z \gamma_o Ah = \pi \tau_0 \gamma_o Ah \sin(\delta) = \pi G'' \gamma_o^2 V$

Shear Strain

Shear Stress
Frequency-Domain Stress-Strain Relation

\[ \tau(t) = G' \gamma(t) + G'' \dot{\gamma}(t) / \bar{\omega} \]

Apply Fourier Transform:

\[ \tau(\bar{\omega}) = G' \gamma(\bar{\omega}) + G'' i \bar{\omega} \gamma(\bar{\omega}) / \bar{\omega} \]

\[ \tau(\bar{\omega}) = \left[ G' + iG'' \right] \gamma(\bar{\omega}) \]

\[ \tau(\bar{\omega}) = G' \left[ 1 + i \eta \right] \gamma(\bar{\omega}) \]

Complex Shear Modulus:

\[ G^*(\bar{\omega}) = \frac{\tau(\bar{\omega})}{\gamma(\bar{\omega})} = G' \left[ 1 + i \eta \right] \]

\[ \tau(\bar{\omega}) = G^*(\bar{\omega}) \gamma(\bar{\omega}) \]

Compact Stress-Strain Relation for Viscoelastic Materials
Dependence of Storage and Loss Moduli on Temperature and Frequency for Typical Viscoelastic Damper

![Graph showing the dependence of storage and loss moduli on temperature and frequency for a typical viscoelastic damper.](image)
Dependence of Loss Factor on Temperature and Frequency for Typical Viscoelastic Damper

Increasing Temperature

Loss Factor

Frequency (Hz)
Actual Hysteretic Behavior of Viscoelastic Damper

Harmonic Loading

Seismic Loading
Advantages of Viscoelastic Dampers

• High reliability
• *May be able to use* linear analysis
• Somewhat lower cost
Disadvantages of Viscoelastic Dampers

- Strong Temperature Dependence
- Lower Force and Displacement Capacity
- Not Force Limited
- Necessity for nonlinear analysis in most practical cases (as it has been shown that it is generally not possible to add enough damping to eliminate all inelastic response)
Outline: Part II

• Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
• Models for Velocity-Dependent Dampers
• Effects of Linkage Flexibility
• Displacement-Dependent Damping Systems: Steel Plate Dampers, Unbonded Brace Dampers, and Friction Dampers
• Concept of Equivalent Viscous Damping
• Modeling Considerations for Structures with Passive Damping Systems
Modeling Viscous Dampers: Simple Dashpot

Useful For:
Fluid Dampers with Zero Storage Stiffness

This Model Ignores Temperature Dependence
Modeling Linear Viscous/Viscoelastic Dampers: Kelvin Model

\[ P(t) = K_D u(t) + C_D \dot{u}(t) \]

Useful For:
Viscoelastic Dampers and Fluid Dampers with Storage Stiffness and Weak Frequency Dependence.

This Model Ignores Temperature Dependence
Kelvin Model (Continued)

\[ P(t) = K_D u(t) + C_d \dot{u}(t) \]

Apply Fourier Transform:

\[ P(\omega) = \left[ K_D + i\omega C_d \right] u(\omega) \]

Complex Stiffness:

\[ K^*(\omega) = K_D + i\omega C_d \]

Storage Stiffness:

\[ K_S(\omega) = \Re \left[ K^*(\omega) \right] = K_D \]

Loss Stiffness:

\[ K_L(\omega) = \Im \left[ K^*(\omega) \right] = C_D \omega \]

Damping Coefficient:

\[ C(\omega) = \frac{K_L(\omega)}{\omega} = C_D \]

FEMA

Instructional Material Complementing FEMA 451, Design Examples

Passive Energy Dissipation 15 – 6 - 71
Kelvin Model (Continued)

\[
C_D = \frac{1}{\omega}
\]

\[
K_D = \frac{1}{\omega^2}
\]

Kelvin Model

\[
C(\bar{\omega}) = C_D
\]

\[
K_S(\bar{\omega}) = K_D
\]

Equivalent Kelvin Model
Modeling Linear Viscous/Viscoelastic Dampers: Maxwell Model

\[ P(t) + \frac{C_D}{K_D} \dot{P}(t) = C_D \ddot{u}(t) \]

Useful For:
Viscoelastic Dampers and Fluid Dampers with Strong Frequency Dependence.
This Model Ignores Temperature Dependence.
Maxwell Model (Continued)

\[ P(t) + \frac{C_D}{K_D} \dot{P}(t) = C_d \dot{u}(t) \]

Apply Fourier Transform:

\[ P(\overline{\omega}) + i\overline{\omega} \frac{C_D}{K_D} P(\overline{\omega}) = i\overline{\omega} C_d u(\overline{\omega}) \]

Complex Stiffness:

\[ K^*(\overline{\omega}) = \frac{C_D \lambda \overline{\omega}^2}{1 + \lambda^2 \overline{\omega}^2} + i \frac{C_D \overline{\omega}}{1 + \lambda^2 \overline{\omega}^2} \]

Relaxation Time: \( \lambda = C_D/K_D \)

Storage Stiffness:

\[ K_S(\overline{\omega}) = \Re\left[K^*(\overline{\omega})\right] = \frac{K_D \lambda^2 \overline{\omega}^2}{1 + \lambda^2 \overline{\omega}^2} \]

Loss Stiffness:

\[ K_L(\overline{\omega}) = \Im\left[K^*(\overline{\omega})\right] = \frac{C_D \overline{\omega}}{1 + \lambda^2 \overline{\omega}^2} \]

Damping Coefficient:

\[ C(\overline{\omega}) = \frac{K_S(\overline{\omega})}{\overline{\omega}} = \frac{C_D}{1 + \lambda^2 \overline{\omega}^2} \]
Maxwell Model Parameters from Experimental Testing of Fluid Viscous Damper

![Graph showing storage stiffness vs. excitation frequency](image)

- **Storage Stiffness** (lb/in)
- **Excitation Frequency** (Hz)

- **KD** indicates the stiffness parameter.

Instructional Material Complementing FEMA 451, Design Examples
Maxwell Model Parameters from Experimental Testing of Fluid Viscous Damper

![Graph showing the relationship between excitation frequency and damping coefficient. The graph indicates a decrease in damping coefficient with an increase in excitation frequency. The damping coefficient, labeled as $C_D$, is shown to be highest at low excitation frequencies and decreases as the frequency increases.]
Maxwell Model (Continued)

Note:
- If $K_D$ is very large, $\lambda$ is very small, $K_S$ is small and $C = C_D$
- If $C_D$ is very small, $\lambda$ is very small, $K_S$ is small and $C = C_D$
- If $K_D$ is very small, $\lambda$ is very large, $C$ is small and $K_S = K_D$.

\[
C(\bar{\omega}) = \frac{C_D}{1 + \lambda^2 \bar{\omega}^2}
\]

\[
K_S(\bar{\omega}) = \frac{K_D \lambda^2 \bar{\omega}^2}{1 + \lambda^2 \bar{\omega}^2}
\]
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Effect of Linkage Flexibility on Damper Effectiveness

Because the damper is always in series with the linkage, the damper-brace assembly acts like a Maxwell model.

Hence, the effectiveness of the damper is reduced. The degree of lost effectiveness is a function of the structural properties and the loading frequency.

\[ K_{Brace,\text{Effective}} = 2 \frac{AE}{L} \cos^2 \theta \]
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Steel Plate Dampers

(Added Damping and Stiffness System - ADAS)
Implementation of ADAS System

Wells Fargo Bank, San Francisco, CA
- Seismic Retrofit of Two-Story Nonductile Concrete Frame; Constructed in 1967
- 7 Dampers Within Chevron Bracing Installed in 1992
- Yield Force Per Damper: 150 kips
Hysteretic Behavior of ADAS Device

ADAS Device
(Tsai et al. 1993)

Experimental Response (Static)
(Source: Tsai et al. 1993)
Ideal Hysteretic Behavior of ADAS Damper
(SAP2000 and ETABS Implementation)

\[ F = \beta k D + (1 - \beta) F_y Z \]

\[
\dot{Z} = \frac{k}{F_y} \begin{cases} 
\dot{D} \left(1 - \left|Z\right|^\alpha \right) & \text{if } \dot{D} \cdot Z > 0 \\
\dot{D} & \text{otherwise}
\end{cases}
\]

\[
Z \text{ is a Path Dependency Parameter}
\]
Parameters of Mathematical Model of ADAS Damper

\[ k = \frac{n(2 + a / b)EI_b}{L^3} \]

\[ F_y = \frac{nf_ybt^3}{4L} \]

- \( n \) = Number of plates
- \( f_y \) = Yield force of each plate
- \( I_b \) = Second moment of area of each plate at b (i.e., at top of plate)
Unbonded Brace Damper

Steel Brace (yielding core) (coated with debonding chemicals)

Stiff Shell Prevents Buckling of Core

Concrete
Implementation of Unbonded Brace Damper

Plant and Environmental Sciences Replacement Facility

- New Three-Story Building on UC Davis Campus
- First Building in USA to Use Unbonded Brace Damper
- 132 Unbonded Braced Frames with Diagonal or Chevron Brace Installation
- Cost of Dampers = 0.5% of Building Cost

Hysteretic Behavior of Unbonded Brace Damper

- Yielding steel core
- Encasing mortar
- 'Unbonding' material between steel core and mortar
- Steel tube

Axial force-displacement behavior
Testing of Unbonded Brace Damper

Testing Performed at UC Berkeley

Typical Hysteresis Loops from Cyclic Testing
Advantages of ADAS System and Unbonded Brace Damper

• Force-Limited
• Easy to construct
• Relatively Inexpensive
• Adds both “Damping” and Stiffness
Disadvantages of ADAS System and Unbonded Brace Damper

• Must be Replaced after Major Earthquake
• Highly Nonlinear Behavior
• Adds Stiffness to System
• Undesirable Residual Deformations Possible
Friction Dampers: Slotted-Bolted Damper

HARDENED WASHER
8-EH-112 SOLON
COMPRESSION WASHERS
UNDER NUT

1/8" TH. BRASS INSERT PLATES

DIRECT TENSION INDICATOR WASHER
(DTI), UNDER HEAD

1/2" DIA. A325 BOLT, 3-1/2" LONG

ALL PLATES ARE OF 5/8" TH. A36 STEEL

WELD
9/16"x3-1/2" LONG SLOT

Pall Friction Damper
Sumitomo Friction Damper
(Sumitomo Metal Industries, Japan)
Pall Cross-Bracing Friction Damper

Figure 3. Cross Section
Figure 4. Typical Braced Bay

Interior of Webster Library at Concordia University, Montreal, Canada
Implementation of Pall Friction Damper

McConnel Library at Concordia University, Montreal, Canada
- Two Interconnected Buildings of 6 and 10 Stories
- RC Frames with Flat Slabs
- 143 Cross-Bracing Friction Dampers Installed in 1987
- 60 Dampers Exposed for Aesthetics
Hysteretic Behavior of Slotted-Bolted Friction Damper
**Ideal Hysteretic Behavior of Friction Damper**

The diagram illustrates the ideal hysteretic behavior of a friction damper, with the force-displacement relationship depicted. The equation for the force ($F_D$) is given by:

$$F_D = N\mu \left| \dot{u}(t) \right|$$

Alternatively,

$$F_D = N\mu \text{sgn}[\dot{u}(t)]$$

where:
- $N$ is the normal force,
- $\mu$ is the coefficient of friction,
- $\dot{u}(t)$ is the velocity of displacement,
- $\text{sgn}()$ is the sign function.

The force is linearly dependent on the velocity, with the force being zero when the velocity is zero. The diagram shows the force in kips on the y-axis and displacement in inches on the x-axis, with the force-displacement relationship for both positive and negative displacements.
Advantages of Friction Dampers

• Force-Limited
• Easy to construct
• Relatively Inexpensive
Disadvantages of Friction Dampers

• May be Difficult to Maintain over Time
• Highly Nonlinear Behavior
• Adds Large Initial Stiffness to System
• Undesirable Residual Deformations Possible
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- Models for Velocity-Dependent Dampers
- Effects of Linkage Flexibility
- Displacement-Dependent Damping Systems: Steel Plate Dampers, Unbonded Brace Dampers, and Friction Dampers
- Concept of Equivalent Viscous Damping
- Modeling Considerations for Structures with Passive Damping Systems
Equivalent Viscous Damping: Damping System with Inelastic or Friction Behavior

\[ \xi_H = \frac{E_H}{4\pi E_s} \]

Note: Computed damping ratio is displacement-dependent
Effect of Inelastic System Post-Yielding Stiffness on Equivalent Viscous Damping

\[ \xi_H = \frac{2(\mu - 1)(1 - \alpha)}{\pi \mu (1 + \mu \alpha - \alpha)} \]

Note: May be Modified (κ) for Other (less Robust) Hysteretic Behavior
Equivalent Viscous Damping: “Equivalent” System with Linear Viscous Damper

\[ C = 2m \omega \xi_H \]

\( E_S \) and \( \omega \) are based on Secant Stiffness of Inelastic System
Equivalent Viscous Damping: Caution!

• It is not possible, on a device level, to “replace” a displacement-dependent device (e.g. a Friction Damper) with a velocity-dependent device (e.g. a Fluid Damper).

• Some simplified procedures allow such replacement on a structural level, wherein a “smeared” equivalent viscous damping ratio is found for the whole structure. This approach is marginally useful for preliminary design, and should not be used under any circumstances for final design.
Outline: Part II

• Velocity-Dependent Damping Systems: Fluid Dampers and Viscoelastic Dampers
• Models for Velocity-Dependent Dampers
• Effects of Linkage Flexibility
• Displacement-Dependent Damping Systems: Steel Plate Dampers, Unbonded Brace Dampers, and Friction Dampers
• Concept of Equivalent Viscous Damping
• Modeling Considerations for Structures with Passive Damping Systems
Modeling Considerations for Structures with Passive Energy Dissipation Devices

- Damping is almost always nonclassical (Damping matrix is not proportional to stiffness and/or mass)

- For seismic applications, system response is usually partially inelastic

- For seismic applications, viscous damper behavior is typically nonlinear (velocity exponents in the range of 0.5 to 0.8)

Conclusion: This is a **NONLINEAR** analysis problem!
Outline: Part III

• Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
• Representations of Damping
• Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
• Summary of MDOF Analysis Procedures
Seismic Analysis of Structures with Passive Energy Dissipation Systems

Linear Behavior?

Yes
(implies viscous or viscoelastic behavior)

Classical Damping?

No

Nonlinear Response-History Analysis

Yes

Complex Modal Response Spectrum Analysis
or
(Complex Modal) Response-History Analysis

No

Modal Response Spectrum Analysis
or
(Modal) Response-History Analysis
Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems

\[ M\ddot{v}(t) + C_I \dot{v}(t) + C_A \dot{v}(t) + F_S(t) = -MR\ddot{v}_g(t) \]

- **Inherent Damping:** Linear
- **Added Viscous Damping:** Linear or **Nonlinear**
- **Restoring Force:** (May include Added Devices) Linear or **Nonlinear**

\[ C_A \neq f(\omega) \]
MDOF Solution Techniques

\[ M \ddot{v}(t) + C_I \dot{v}(t) + C_A \dot{v}(t) + F_S(t) = -MR\ddot{g}(t) \]

Explicit integration of fully coupled equations:

- Treat \( C_I \) as Rayleigh damping and model \( C_A \) explicitly.

- Use Newmark solver (with iteration) to solve full set of coupled equations.

**System may be linear or nonlinear.**
MDOF Solution Techniques

\[ M\ddot{v}(t) + C_I \dot{v}(t) + C_A \dot{v}(t) + F_S(t) = -MR\ddot{v}_g(t) \]

**Fast Nonlinear Analysis:**
Treat \( C_I \) as modal damping and model \( C_A \) explicitly. Move \( C_A \) (and any other nonlinear terms) to right-hand side. Left-hand side may be uncoupled by Ritz Vectors. Iterate on unbalanced right-hand side forces.

*System may be linear or nonlinear.*
Fast Nonlinear Analysis

\[ M \ddot{v}(t) + C_I \dot{v}(t) + C_A \dot{v}(t) + K_E v(t) + F_H(t) = -M R \ddot{v}_g(t) \]

Move Added Damper Forces and Nonlinear Forces to RHS:

\[ M \ddot{v}(t) + C_I \dot{v}(t) + K_E v(t) = -M R \ddot{v}_g(t) - F_H(t) - C_A \dot{v}(t) \]

Linear Terms

Nonlinear Terms

Transform Coordinates: \( v(t) = \Phi y(t) \)

Apply Transformation:

\[ \tilde{M} \ddot{y}(t) + \tilde{C}_I \dot{y}(t) + \tilde{K}_E y(t) = -\Phi^T M R \ddot{v}_g(t) - \Phi^T F_H(t) - \tilde{C}_A \dot{y}(t) \]

Uncoupled

Coupled

Orthogonal basis of Ritz vectors: Number of vectors \(<< N\)
Fast Nonlinear Analysis

Relative Solution Speed:

FNA/Full Integration

Nonlinear DOF / Total DOF

0.0 1.0

1.0 10.0
MDOF Solution Techniques

\[ M\ddot{v}(t) + C_I \dot{v}(t) + C_A \dot{v}(t) + F_S(t) = -MR\ddot{v}_g(t) \]

Explicit integration or response spectrum analysis of first few uncoupled modal equations:

- Treat \( C_I \) as modal damping or Rayleigh damping
- Use Modal Strain Energy method to represent \( C_A \) as modal damping ratios.

System must be linear.
Applicable only to viscous (or viscoelastic) damping systems.
Outline: Part III

• Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
• Representations of Damping
• Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
• Summary of MDOF Analysis Procedures
Modal Damping Ratios

\[ M\ddot{v} + C\dot{v} + K\dot{v} = -MR\ddot{v}_g \]

\[ v = \Phi y \]

\[ \ddot{y}_i + 2\xi_i\omega_i \dot{y}_i + \omega_i^2 y_i = \Gamma_i \ddot{v}_g \]

Specify modal damping values directly
Modal Superposition Damping

\[ C = M \left[ \sum_{i=1}^{n} \frac{2 \xi_i \omega_i}{\phi_i^T M \phi_i} \phi_i^T \phi_i \right] M \]

Skyhook

Artificial Coupling

Note: There is no need to develop \( C \) explicitly.
Rayleigh Proportional Damping

\[ C_R = \alpha M + \beta K \]
Derivation of Modal Strain Energy Method

Floor Displacement

\[ \phi_{2,1} \]
\[ \phi_{2,2} \]
\[ \phi_{2,3} \]
\[ \phi_{2,4} \]

Damper Deformation

\[ \phi_{2,1} - \phi_{2,2} \]
\[ \phi_{2,2} - \phi_{2,3} \]
\[ \phi_{2,3} - \phi_{2,4} \]
\[ \phi_{2,4} \]
Derivation of Modal Strain Energy Method

\[ E_{D,i,\text{story}k} = \pi \omega_i C_k (\phi_{i,k} - \phi_{i,k-1})^2 \]

\[ E_{S,i} = \frac{1}{2} \phi_i^T K \phi_i = \frac{1}{2} \omega_i^2 \phi_i^T M \phi_i \]

\[ \xi_i = \frac{\sum_{k=1}^{n \text{ stories}} E_{D,i,\text{story}k}}{4\pi E_{S,i}} \]
Derivation of Modal Strain Energy Method

\[ \xi_i = \frac{\sum_{k=1}^{N \text{ stories}} C_k (\phi_{i,k} - \phi_{i,k-1})^2}{2 \omega_i \phi_i^T M \phi_i} \]

\[ \xi_i = \frac{\phi_i^T C_A \phi_i}{2 \omega_i \phi_i^T M \phi_i} = \frac{\phi_i^T C_A \phi_i}{2 m_i^* \omega_i} \]

Note: IF \( C_A \) is diagonalized by \( \Phi \), THEN

\[ \xi_i = \frac{c_i^*}{2 m_i^* \omega_i} \]
Modal Strain Energy Damping Ratio

\[ \xi_i = \frac{\phi_i^T C_A \phi_i}{2m_i \omega_i} \]

Note: \( \phi \) is the Undamped Mode Shape

The Modal Strain Energy Method is approximate if the structure is non-classically damped since the undamped and damped mode shapes are different.
Outline: Part III

• Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
• Representations of Damping
• Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
• Summary of MDOF Analysis Procedures
Example: Application of Modal Strain Energy Method

\[ m = 1.0 \text{ k-sec}^2/\text{in.} \]

\[ m = 1.5 \]
\[ m = 1.5 \]
\[ m = 1.5 \]
\[ m = 1.5 \]

\[ k = 200 \text{ k/in.} \]
\[ k = 250 \]
\[ k = 300 \]
\[ k = 350 \]
\[ k = 400 \]

Proportional Damping

\[ c = 10.0 \text{ k-sec/in.} \]
\[ c = 12.5 \]
\[ c = 15.0 \]
\[ c = 17.5 \]
\[ c = 20.0 \]
Modal Damping Ratios from Modal Strain Energy Method for *Proportional* Damping Distribution

<table>
<thead>
<tr>
<th>Frequency (rad/sec)</th>
<th>Damping Ratio, $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.54</td>
<td>0.113</td>
</tr>
<tr>
<td>12.1</td>
<td>0.302</td>
</tr>
<tr>
<td>18.5</td>
<td>0.462</td>
</tr>
<tr>
<td>23.0</td>
<td>0.575</td>
</tr>
<tr>
<td>27.6</td>
<td>0.690</td>
</tr>
</tbody>
</table>

![Graph](image-url)
Example: Application of Modal Strain Energy Method

$m = 1.0 \text{ k-sec}^2/\text{in.}$

$m = 1.5$

$m = 1.5$

$m = 1.5$

$m = 1.5$

$k = 200 \text{ k/in.}$

$k = 250$

$k = 300$

$k = 350$

$k = 400$

$c = 10 \text{ k-sec/in.}$

$c = 10$

$c = 20$

$c = 30$
### Modal Damping Ratios from Modal Strain Energy Method for Nearly Proportional Damping Distribution

<table>
<thead>
<tr>
<th>Frequency (rad/sec)</th>
<th>Damping Ratio, $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.54</td>
<td>0.123</td>
</tr>
<tr>
<td>12.1</td>
<td>0.318</td>
</tr>
<tr>
<td>18.5</td>
<td>0.455</td>
</tr>
<tr>
<td>23.0</td>
<td>0.557</td>
</tr>
<tr>
<td>27.6</td>
<td>0.702</td>
</tr>
</tbody>
</table>

![Graph showing damping ratios for different frequencies]
Example: Application of Modal Strain Energy Method

\[ m = 1.0 \text{ k-sec}^2/\text{in.} \]

\[ m = 1.5 \]
\[ m = 1.5 \]
\[ m = 1.5 \]
\[ m = 1.5 \]

\[ k = 200 \text{ k/in.} \]
\[ k = 250 \]
\[ k = 300 \]
\[ k = 350 \]
\[ k = 400 \]

Nonproportional Damping

\[ c = 20 \text{ k-sec/in} \]
\[ c = 30 \]
Modal Damping Ratios from Modal Strain Energy Method for *Nonproportional* Damping Distribution

<table>
<thead>
<tr>
<th>Frequency (rad/sec)</th>
<th>Damping Ratio $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.54</td>
<td>0.089</td>
</tr>
<tr>
<td>12.1</td>
<td>0.144</td>
</tr>
<tr>
<td>18.5</td>
<td>0.134</td>
</tr>
<tr>
<td>23.0</td>
<td>0.194</td>
</tr>
<tr>
<td>27.6</td>
<td>0.514</td>
</tr>
</tbody>
</table>
Modal Superposition Damping can be used to construct the damping matrix from the modal damping ratios obtained via the Modal Strain Energy Method.
Comparison of Actual Damping Matrix and Damping Matrix Obtained from MSE Damping Ratios

Proportional Damping

- $c = 10.0 \text{ k-sec/in.}$

- $c = 12.5$

- $c = 15.0$

- $c = 17.5$

- $c = 20.0$

Actual Damping Matrix

$$C_A = \begin{bmatrix}
10.0 & -10.0 & 0 & 0 & 0 \\
-10.0 & 22.5 & -12.5 & 0 & 0 \\
0 & -12.5 & 27.5 & -15.0 & 0 \\
0 & 0 & -15.0 & 32.5 & -17.5 \\
0 & 0 & 0 & -17.5 & 37.5
\end{bmatrix}$$

Modal Superposition Damping Matrix Using MSE Damping Ratios

$$C = \begin{bmatrix}
10.0 & -10.0 & 0 & 0 & 0 \\
-10.0 & 22.5 & -12.5 & 0 & 0 \\
0 & -12.5 & 27.5 & -15.0 & 0 \\
0 & 0 & -15.0 & 32.5 & -17.5 \\
0 & 0 & 0 & -17.5 & 37.5
\end{bmatrix}$$
Comparison of Actual Damping Matrix and Damping Matrix Obtained from MSE Damping Ratios

Nearly Proportional Damping

\[
\begin{bmatrix}
10.0 & -10.0 & 0 & 0 & 0 \\
-10.0 & 20.0 & -10.0 & 0 & 0 \\
0 & -10.0 & 20.0 & -10.0 & 0 \\
0 & 0 & -10.0 & 30.0 & -20.0 \\
0 & 0 & 0 & -20.0 & 50.0
\end{bmatrix}
\]

Actual Damping Matrix

\[
\begin{bmatrix}
10.0 & -9.66 & -0.166 & -0.228 & -0.010 \\
-9.66 & 22.0 & -12.2 & -0.169 & -0.422 \\
-0.166 & -12.2 & 27.3 & -15.1 & -0.731 \\
-0.228 & -0.169 & -15.1 & 33.1 & -17.8 \\
-0.010 & -0.422 & -0.731 & -17.8 & 37.5
\end{bmatrix}
\]

Modal Superposition Damping Matrix Using MSE Damping Ratios
Comparison of Actual Damping Matrix and Damping Matrix Obtained from MSE Damping Ratios

Nonproportional Damping

Actual Damping Matrix

\[
C_A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 20.0 & -20.0 \\
0 & 0 & 0 & -20.0 & 50.0
\end{bmatrix}
\]

Modal Superposition Damping Matrix Using MSE Damping Ratios

\[
C = \begin{bmatrix}
3.65 & -2.96 & 0.456 & -1.098 & 0.066 \\
-2.96 & 8.27 & -5.92 & 2.72 & -2.07 \\
0.456 & -5.92 & 13.4 & -10.9 & 6.21 \\
-1.098 & 2.72 & -10.9 & 21.9 & -15.1 \\
0.066 & -2.07 & 6.21 & -15.1 & 20.9
\end{bmatrix}
\]

$c = 20.0$ k-sec/in.

$c = 30.0$
Example: Seismic Analysis of a Structure with Nonproportional Damping

- Discrete Damping vs Rayleigh Damping
- Discrete Damping: Rigid vs Flexible Braces

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<td>0.514</td>
</tr>
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Damping ratios in modes 1 and 4 used to construct Rayleigh damping matrix.
Computing Rayleigh Damping Proportionality Factors (Using NONLIN Pro)
Example: Discrete (Stiff Braces) vs Rayleigh Damping

- **Discrete Dampers**
- **Rayleigh Damping**

![Graph showing peak roof displacement vs first mode damping](image-url)

- Very Stiff Braces
- $c$
- $1.5c$
Example: Discrete (Stiff Braces) vs Rayleigh Damping

- Discrete Dampers
- Rayleigh Damping: Inertial
- Rayleigh Damping: Columns

Very Stiff Braces

Displacement

$c$

$1.5c$
Example: Effect of Brace Stiffness
(Discrete Damping Model)

![Graph showing the effect of brace stiffness on peak roof displacement. The graph plots peak roof displacement in inches against first mode damping as a percentage of critical damping. Two lines represent different brace stiffness conditions: very stiff and reasonably stiff. The graph indicates that lower damping results in lower peak roof displacement.](image-url)
Example: Effect of Brace Stiffness (Discrete Damping Model)
Example: Discrete (Flexible Braces) vs Rayleigh Damping

- Graph showing the relationship between peak roof displacement and first mode damping.
- Discrete Model with Flexible Braces vs Rayleigh Damping.
- The graph indicates a decrease in peak roof displacement as damping increases.
- The inset diagram shows a structural model with flexible braces and labels such as $c$ and $1.5c$.
Example: Discrete (Flexible Braces) vs Rayleigh Damping

- Peak Base Shear, Kips (from Inertial Forces)
- First Mode Damping, % Critical

- Discrete Model with Flexible Braces
- Rayleigh Damping

Example: Discrete (Flexible Braces) vs Rayleigh Damping

- c
- 1.5c
Example: Effect of Brace Stiffness on Peak Story Shear Forces

STIFF BRACES

FLEX BRACES
Outline: Part III

• Seismic Analysis of MDOF Structures with Passive Energy Dissipation Systems
• Representations of Damping
• Examples: Application of Modal Strain Energy Method and Non-Classical Damping Analysis
• Summary of MDOF Analysis Procedures
Summary: MDOF Analysis Procedures (Linear Systems and Linear Dampers)

• Use discrete damper elements and explicitly include these dampers in the system damping matrix. Perform response history analysis of full system. **Preferred.**

• Use discrete damper elements to estimate modal damping ratios and use these damping ratios in modal response history or modal response spectrum analysis. **Dangerous.**

• Use discrete damper elements to estimate modal damping ratios and use these damping ratios in a response history analysis which incorporates Rayleigh proportional damping. **Dangerous.**
Summary: MDOF Analysis Procedures
(Linear Systems with Nonlinear Dampers)

• Use discrete damper elements and explicitly include these dampers in the system damping matrix. Perform response history analysis of full system. **Preferred.**

• Replace nonlinear dampers with “equivalent energy” based linear dampers, and then use these equivalent dampers in the system damping matrix. Perform response history analysis of full system. **Very Dangerous.**

• Replace nonlinear dampers with “equivalent energy” based linear dampers, use modal strain energy approach to estimate modal damping ratios, and then perform response spectrum or modal response history analysis. **Very Dangerous.**
Summary: MDOF Analysis Procedures (Nonlinear Systems with Nonlinear Dampers)

• Use discrete damper elements and explicitly include these dampers in the system damping matrix. Explicitly model inelastic behavior in superstructure. Perform response history analysis of full system. Preferred.

• Replace nonlinear dampers with “equivalent energy” based linear dampers and use modal strain energy approach to estimate modal damping ratios. Use pushover analysis to represent inelastic behavior in superstructure. Use capacity-demand spectrum approach to estimate system deformations. Do This at Your Own Risk!
Outline: Part IV

• MDOF Solution Using Complex Modal Analysis
• Example: Damped Mode Shapes and Frequencies
• An Unexpected Effect of Passive Damping
• Modeling Dampers in Computer Software
• Guidelines and Code-Related Documents for Passive Energy Dissipation Systems
MDOF Solution for Non-Classically Damped Structures Using Complex Modal Analysis

\[
M\ddot{v}(t) + C_I \dot{v}(t) + C_A \dot{v}(t) + F_s(t) = -MR\ddot{v}_g(t)
\]

Modal Analysis using Damped Mode Shapes:

- Treat \(C_I\) as modal damping and model \(C_A\) explicitly.

- Solve by modal superposition using damped (complex) mode shapes and frequencies.

System (dampers and structure) must be linear.
Damped Eigenproblem

\[ M\ddot{v}(t) + C_A \dot{v}(t) + K_v(t) = 0 \]

Assume \( C_I \) is negligible

State Vector: \( Z = \begin{bmatrix} \dot{\mathbf{v}} \\ \mathbf{v} \end{bmatrix} \)

State-Space Transformation: \( \dot{Z} = HZ \)

State Matrix: \( H = \begin{bmatrix} -M^{-1}C_A & -M^{-1}K \\ I & 0 \end{bmatrix} \)
Solution of Damped Eigenproblem

Assume Harmonic Response for n-th mode: \( Z_n = P_n e^{\lambda_n t} \)

Substitute Response into State Space Equation:

\[
P_n \lambda_n = HP_n
\]

Damped Eigenproblem for n-th Mode

\[
P \Lambda = HP
\]

Damped Eigenproblem for All Modes

Eigenvalue Matrix:

\[
\Lambda = \begin{bmatrix} \lambda & \lambda^* \\ \lambda^* & \lambda \end{bmatrix} \quad \lambda = \text{diag} [\lambda_n]
\]

Eigenvector Matrix:

\[
P = \begin{bmatrix} \Phi \lambda & \Phi^* \lambda^* \\ \Phi & \Phi^* \end{bmatrix}
\]
Extracting Modal Damping and Frequency from Complex Eigenvalues

Complex Eigenvalue for Mode n:

\[ \lambda_n = -\xi_n \omega_n \pm i\omega_n \sqrt{1 - \xi_n^2} \]

Modal Frequency:

\[ \omega_n = \left| \lambda_n \right| \]

Modal Damping Ratio:

\[ \xi_n = -\frac{\Re(\lambda_n)}{\left| \lambda_n \right|} \]

Note: \( i = \sqrt{-1} \)

Analogous to Roots of Characteristic Equation for SDOF Damped Free Vibration Problem
Extracting Damped Mode Shapes

\[ P = \begin{bmatrix} \Phi \Lambda & \Phi^* \Lambda^* \\ \Phi & \Phi^* \end{bmatrix} \]

Damped Mode Shapes
Damped Mode Shapes

\[ \phi = \begin{bmatrix} a_1 + ib_1 \\ a_2 + ib_2 \\ a_3 + ib_3 \\ a_4 + ib_4 \end{bmatrix} \]

Real

Imaginary
Outline: Part IV

- MDOF Solution Using Complex Modal Analysis
- Example: Damped Mode Shapes and Frequencies
- An Unexpected Effect of Passive Damping
- Modeling Dampers in Computer Software
- Guidelines and Code-Related Documents for Passive Energy Dissipation Systems
Example: Damped Mode Shapes and Frequencies

$m = 1.0 \text{ k-sec}^2/\text{in.}$

$m = 1.5$

$k = 200 \text{ k/in.}$

$m = 1.5$

$k = 250$

$m = 1.5$

$k = 300$

$m = 1.5$

$k = 350$

$m = 1.5$

$k = 400$

Nonproportional Damping

$c = 20 \text{ k-sec/in}$

$c = 30$
### Example: Damped Mode Shapes and Frequencies

**System with Non-Classical Damping**

#### Using UNDAMPED MODE SHAPES

<table>
<thead>
<tr>
<th>Frequency (rad/sec)</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.54</td>
</tr>
<tr>
<td>2</td>
<td>12.1</td>
</tr>
<tr>
<td>3</td>
<td>18.4</td>
</tr>
<tr>
<td>4</td>
<td>23.0</td>
</tr>
<tr>
<td>5</td>
<td>27.6</td>
</tr>
</tbody>
</table>

#### Using DAMPED MODE SHAPES*

<table>
<thead>
<tr>
<th>Frequency (rad/sec)</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.58</td>
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<tr>
<td>2</td>
<td>12.3</td>
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<td>3</td>
<td>18.9</td>
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<tr>
<td>4</td>
<td>24.0</td>
</tr>
<tr>
<td>5</td>
<td>25.1</td>
</tr>
</tbody>
</table>

*Table is for model with VERY STIFF braces.

---

Obtained from MSE Method

**Significant Differences in Higher Mode Damping Ratios**
Example: Damped Mode Shapes and Frequencies
System with Non-Classical Damping

Mode = 1
\[ \omega = 4.58 \]
\[ \xi = 0.089 \]

System with Non-Classical Damping

Real Component of Mode Shape

Imaginary Component of Mode Shape
Example: Damped Mode Shapes and Frequencies

System with Non-Classical Damping

\[ \omega = 12.34 \]
\[ \xi = 0.141 \]

Mode = 2

Level 5
Level 4
Level 3
Level 2
Level 1
Outline: Part IV

• MDOF Solution Using Complex Modal Analysis
• Example: Damped Mode Shapes and Frequencies
• An Unexpected Effect of Passive Damping
• Modeling Dampers in Computer Software
• Guidelines and Code-Related Documents for Passive Energy Dissipation Systems
An Unexpected Effect of Passive Damping

The larger the damping coefficient $C$, the smaller the damping ratio $\xi$.

Why?

Note: Occurs for toggle-braced systems only.

Huntington Tower
- 111 Huntington Ave, Boston, MA
- New 38-story steel-framed building
- 100 Direct-acting and toggle-brace dampers
- 1300 kN (292 kips), +/- 101 mm (+/- 4 in.)
- Dampers suppress wind vibration
Toggle Brace Deployment

Huntington Tower
Example: Toggle Brace Damping System

Units: inches
Methods of Analysis Used to Determine Damping Ratio

• Energy Ratios for Steady-State Harmonic Loading: $\xi = \frac{E_D}{4\pi E_S}$

• Modal Strain Energy

• Free Vibration Log Decrement

• Damped Eigenproblem

$C = $10 to 40 k-sec/in (increments of 10)

$A = $10 to 100 in$^2$ (increments of 10)
Computed Damping Ratios for System With A = 10

![Graph showing computed damping ratios for a system with A = 10. The graph plots Damping Ratio against Damping Coefficient (k-sec/in). The graph includes lines for Energy Ratio, Modal Strain Energy, Log Decrement, and Damped Eigenproblem.]
Computed Damping Ratios for System With A = 20

![Graph showing computed damping ratios for a system with A = 20. The graph plots damping ratio against damping coefficient (k-sec/in). The graph includes multiple curves representing different types of damping: energy ratio, modal strain energy, log decrement, and damped eigenproblem. Each curve is color-coded for easy differentiation.]
Computed Damping Ratios for System With A = 30

A30

- Energy Ratio
- Modal Strain Energy
- Log Decrement
- Damped Eigenproblem

Damping Ratio vs. Damping Coefficient (k-sec/in)
Computed Damping Ratios for System With $A = 50$

![Graph showing computed damping ratios for a system with $A = 50$. The x-axis represents the damping coefficient (k-sec/in), and the y-axis represents the damping ratio. The graph includes lines for energy ratio, modal strain energy, log decrement, and damped eigenproblem.]
Why Does Damping Ratio Reduce for Low Brace Area/Damping Coefficient Ratios?

Displacement in Damper is *Out-of-Phase* with Displacement at DOF 1
Phase Difference Between Damper Displacement and Frame Displacement

A=10
C=40
A/C=0.25

A=50
C=40
A/C=1.25
Damped Mode Shapes for System With A=20 in²

C=0

C=10

C=20

C=30

C=40

C=50

U₁, U₂, U₃
Interim Summary Related to Modeling and Analysis (1)

- Viscously damped systems are very effective in reducing damaging deformations in structures.

- With minor exceptions, viscously damped systems are non-classical, and *must* be modeled explicitly using dynamic time history analysis.

- Avoid the use of the Modal Strain Energy method (it may provide unconservative results)
Interim Summary Related to Modeling and Analysis (2)

• Damped mode shapes provide phase angle information that is essential in assessing and improving the efficiency of viscously damped systems. This is particularly true for linkage systems (e.g. toggle-braced systems).

• If damped eigenproblem analysis procedures are not available, use overlayed response history plots of damper displacement and interstory displacement to assess damper efficiency. (This would be required for nonlinear viscously damped systems.)
Outline: Part IV

• MDOF Solution Using Complex Modal Analysis
• Example: Damped Mode Shapes and Frequencies
• An Unexpected Effect of Passive Damping
• Modeling Dampers in Computer Software
• Guidelines and Code-Related Documents for Passive Energy Dissipation Systems
## Computer Software Analysis Capabilities

<table>
<thead>
<tr>
<th>System Type</th>
<th>SAP2000; ETABS</th>
<th>DRAIN</th>
<th>RAM Perform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Viscous Fluid Dampers</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Nonlinear Viscous Fluid Dampers</td>
<td>Yes</td>
<td>NO</td>
<td>Yes*</td>
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<tr>
<td>Viscoelastic Dampers</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>ADAS Type Systems</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Unbonded Brace Systems</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Friction Systems</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>General System Yielding</td>
<td>Pending</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

*Piecewise Linear
Modeling Linear Viscous Dampers in DRAIN

Use a Type-1 truss bar element with stiffness proportional damping:

\[ K = \frac{AE}{L}, \quad C = \beta K \]

For dampers with low stiffness:
Set \( A = L, \ E = 0.01 \) and \( \beta = \frac{C_{\text{Damper}}}{0.01} \)

Result:
\[ K = 0.01, \quad C = C_{\text{Damper}} \]
\[ F = C\ddot{u} = \beta K\ddot{u} = C_{\text{Damper}}\ddot{u} \]
Dampers may be similarly modeled using the zero-length “Type-4” connection element.

Nodes $j$ and $m$ have the same coordinates.
Modeling Viscous/Viscoelastic Dampers Using the SAP2000 NLLINK Element

The damper is modeled as a Maxwell Element consisting of a linear or nonlinear dashpot in series with a linear spring.

To model a linear viscous dashpot, $K_D$ must be set to a large value, but not too large or convergence will not be achieved. To achieve this, it is recommended that the relaxation time ($\lambda = C_D/K_D$) be an order of magnitude less than the loading time step $\Delta t$. For example, let $K_D = 100C_D/\Delta t$. Sensitivity to $K_D$ should be checked.

SAP2000 often has difficulty converging when nonlinear dampers are used and the velocity exponent is less than 0.4.
Modeling ADAS, Unbonded Brace, and Friction Dampers using the SAP2000 NLLINK Element

\[ F = \beta k D + (1 - \beta) F_y Z \]

\[ \dot{Z} = \frac{k}{F_y} \begin{cases} \dot{D}(1 - |Z|^\alpha) & \text{if } \dot{D}Z > 0 \\ \dot{D} & \text{otherwise} \end{cases} \]

Note: \( Z \) is an internal hysteretic variable with magnitude less than or equal to unity. The yield surface is associated with a magnitude of unity.

For bilinear behavior, use \( \alpha \) of approximately 50. Larger values can produce strange results.
Outline: Part IV

• MDOF Solution Using Complex Modal Analysis
• Example: Damped Mode Shapes and Frequencies
• An Unexpected Effect of Passive Damping
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• Guidelines and Code-Related Documents for Passive Energy Dissipation Systems

- Draft version developed by Energy Dissipation Working Group (EDWG) of Base Isolation Subcommittee of Seismology Committee of SEAONC (Not reviewed/approved by SEAOC; used as basis for 1994 NEHRP Provisions)

- Philosophy: For Design Basis Earthquake (10/50), confine inelastic behavior to energy dissipation devices (EDD); gravity load resisting system to remain elastic

- Established terminology and nomenclature for energy dissipation systems (EDS)

- Classified systems as rate-independent or rate-dependent (included metallic, friction, viscoelastic, and viscous dampers)

- Required at least two vertical lines of dampers in each principal direction of building; dampers to be continuous from the base of the building

- Prescribed analysis and testing procedures

Energy Dissipation Nomenclature

Energy Dissipation Device (EDD)

Energy Dissipation Assembly (EDA)

- Elastic structures with rate-dependent devices: Linear dynamic procedures (response spectrum or response history analysis)

- Inelastic structures or structures with rate-independent devices: Nonlinear dynamic response history analysis

- Prototype tests on full-size specimens (not required if previous tests performed and documented by ICBO)

- General acceptability criteria for energy dissipation systems:
  - Remain stable at design displacements
  - Provide non-decreasing resistance with increasing displacement (for rate-independent systems)
  - Exhibit no degradation under repeated cyclic load at design displ.
  - Have quantifiable engineering parameters

- Independent engineering review panel required to oversee design and testing
1994 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (1 of 4)
Part 1 – Provisions & Part 2 – Commentary (FEMA 222A & 223A)

- Includes Appendix to Chapter 2 entitled: Passive Energy Dissipation Systems

- Material is based on:
  - 1993 draft SEAONC EDWG document
  - Proceedings of ATC 17-1 Seminar on Seismic Isolation, Passive Energy Dissipation, and Active Control (March 1993)
  - Special issue of Earthquake Spectra (August 1993)

- Applicable to wide range of EDD’s; therefore requires EDD performance verification via prototype testing

- Performance objective identical to conventional structural system (i.e., life-safety for design EQ)

- At least two EDD per story in each principal direction, distributed continuously from base to top of building unless adequate performance (drift limits satisfied and member curvature capacities not exceeded) with incomplete vertical distribution can be demonstrated

- Members that transmit damper forces to foundation designed to remain elastic
Analysis/Design Procedure for Linear Viscous Energy Dissipation Systems

\[ V_{\text{min}} = BV = BC_S W \]

- \( V_{\text{min}} \): Minimum base shear for design of structure with EDS
- \( V \): Minimum base shear for design of structure without EDS
- \( B \): Reduction factor to account for energy dissipation provided by EDS
  (based on combined, inherent plus added damping, damping ratio)

**Note:** After publication, it was recognized that this procedure may not be appropriate since it allows reduction in forces due to both inelastic structural response (R-factor) and added damping (B-factor). For yielding structures, added damping will not reduce forces.
Analysis/Design Procedure for EDD’s other than Linear Viscous Dampers

1) **Preliminary Design**: Linear dynamic modal analysis using effective stiffness and damping coefficient of energy dissipation devices. Use B-factor to reduce modal base shears.

\[ k_{D_{\text{eff}}}^D = \frac{|F_D^+| + |F_D^-|}{|\Delta^+| + |\Delta^-|} \]

**Eq. (C2A.3.2.1a)**

Effective Device Stiffness at Design Displacement

\[ c_{eq} = 2m\omega_n \xi_{eq} = \frac{2m\omega_n W_D}{4\pi W_S} = \frac{W_D T}{2\pi^2 \Delta^2} \]

**Eq. (2A.3.2.1)**

Equivalent Device Damping Coefficient

\[ \xi_{combined} = \xi_{str} + \frac{\sum W_D}{4\pi SE} \]

**Eq. (C2A.3.2.1c)**

Combined Equivalent Damping Ratio

2) **Performance Verification**: Nonlinear response history analysis

---

1994 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (3 of 4)
Part 1 – Provisions & Part 2 – Commentary (FEMA 222A & 223A)

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Instructional Material Complementing FEMA 451, Design Examples

Passive Energy Dissipation 15 – 6 - 186
- For nonlinear response-history analysis, mathematical modeling should account for:
  - Plan and vertical spatial distribution of EDD’s
  - Dependence of EDD’s on loading frequency, temperature, sustained loads, nonlinearity, and bilateral loads

- Prototype Tests on at least two full-size EDD’s (unless prior testing has been documented)
  - 200 fully reversed cycles corresponding to wind forces
  - 50 fully reversed cycles corresponding to design earthquake
  - 10 fully reversed cycles corresponding to maximum capable earthquake

- Acceptability criteria from prototype testing of EDD’s:
  - Hysteresis loops have non-negative incremental force-carrying capacities (for rate-independent systems only)
  - Exhibit limited effective stiffness degradation under repeated cyclic load
  - Exhibit limited degradation in energy loss per cycle under repeated cyclic load
  - Have quantifiable engineering parameters
  - Remain stable at design displacements
- Includes an appendix to Chapter 13 entitled: Passive Energy Dissipation

- The appendix in the 1994 NEHRP Provisions was deleted since it was deemed to be insufficient for design and regulation. It was replaced with 3 paragraphs that provide very general guidance on passive energy dissipation systems.
Chapter 9 entitled: Seismic Isolation and Energy Dissipation
(Developed by New Technologies Team under ATC Project 33)

Performance-based document
- Rehabilitation objectives based on desired performance levels for selected hazard levels

Global Structural Performance Levels
- Operational (OP)
- Immediate Occupancy (IO)
- Life-Safety (LS)
- Collapse Prevention (CP)

Hazard levels
- Basic Safety Earthquake 1 (BSE-1): 10/50 event
- Basic Safety Earthquake 2 (BSE-2): 2/50 event (Maximum Considered EQ - MCE)

Rehabilitation Objectives
- Limited Objectives (less than BSO)
- Basic Safety Objective (BSO): LS for BSE-1 and CP for BSE-2
- Enhanced Objectives (more than BSO)

Most Applicable Performance Levels

Applicable Rehabilitation Objectives
- Simplified vs. Systematic Rehabilitation
  - Simplified: For simple structures in areas of low to moderate seismicity
  - Systematic: Considers all elements needed to attain rehabilitation objective

- Systematic Rehabilitation methods of analysis:
  - Linear static procedure (LSP)
  - Linear dynamic procedure (LDP)
  - Nonlinear static procedure (NSP)
  - Nonlinear dynamic procedure (NDP)

  - Coefficient Method
  - Capacity Spectrum Method
• **Basic Principles:**
  – Dampers should be spatially distributed (at each story and on each side of building)
  – Redundancy (at least two dampers along the same line of action; design forces for dampers and damper framing system are reduced as damper redundancy is increased)
  – For BSE-2, dampers and their connections designed to avoid failure (i.e., not weak link)
  – Members that transmit damper forces to foundation designed to remain elastic

• **Classification of EDD’s**
  – Displacement-dependent
  – Velocity-dependent
  – Other (e.g., shape memory alloys and fluid restoring force/damping dampers)

Manufacturing quality control program should be established along with prototype testing programs and independent panel review of system design and testing program
Mathematical Modeling of Displacement-Dependent Devices

\[ F = k_{\text{eff}} D \]

\[ k_{\text{eff}} = \left| \frac{F^+ + F^-}{D^+ + D^-} \right| \]

\[ \beta_{\text{eff}} = \frac{1}{2\pi} \frac{W_D}{k_{\text{eff}} D_{\text{ave}}^2} \]
Mathematical Modeling of Solid Viscoelastic Devices

**Equation (9-22)**

\[ F = k_{eff} D + C \dot{D} \]

**Force in Device**

**Equation (9-23)**

\[ k_{eff} = \frac{|F^+| + |F^-|}{|D^+| + |D^-|} = K' \]

**Effective Stiffness of Device**

**Equation (9-24)**

\[ C = \frac{W_D}{\pi \omega_1 D_{ave}^2} = \frac{K''}{\omega_1} \]

**Damping Coefficient of Device**

**Average Peak Displ.**

**Circular frequency of mode 1**

**Loss Stiffness**

**Storage Stiffness**

**Area**

\[ W_D \]

**Slope**

\[ k_{eff} \]

**Deformation**

**Force**

\[ F^+, F^-, D^+, D^- \]
Mathematical Modeling of Fluid Viscoelastic and Fluid Viscous Devices

Fluid Viscoelastic Devices:

\[ F + \lambda \dot{F} = C \dot{D} \]  
Maxwell Model

Fluid Viscous Devices:

\[ F = C_0 |\dot{D}|^\alpha sgn(\dot{D}) \]  
Eq. (9-25)  
Linear or Nonlinear Dashpot Model

Caution: Only use fluid viscous device model if \( K' > 0 \) for frequencies between 0.5 \( f_1 \) and 2.0 \( f_1 \); Otherwise, use fluid viscoelastic device model.
Pushover Analysis for Structures with EDD’s (Part of NSP)

Performance point without dampers
Performance point with dampers

Reduced Displacement
Reduced Damage

With Viscous Dampers
No dampers

With ADAS Dampers
No dampers

With Friction Dampers
No dampers

With Viscoelastic Dampers
No dampers
Design Process for Velocity-Dependent Dampers using NSP

<table>
<thead>
<tr>
<th>Steps</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>Estimate Target Displacement (performance point)</td>
</tr>
<tr>
<td>2)</td>
<td>Calculate Effective Damping Ratio and Secant Stiffness of building with dampers at Target Displacement</td>
</tr>
<tr>
<td>3)</td>
<td>Use Effective Damping and Secant Stiffness to calculate revised Target Displacement</td>
</tr>
<tr>
<td>4)</td>
<td>Compare Target Displacement from Steps 1 and 4. If within tolerance, stop. Otherwise, return to Step 1.</td>
</tr>
</tbody>
</table>

\[
\beta_{eff} = \beta + \frac{\sum_{j} W_j}{4\pi W_k} \\
W_k = \frac{1}{2} \sum_{i} F_i \delta_i
\]

- \(\beta_{eff}\) = Effective damping ratio of building with dampers at Target Displ.; 
  \(j = \) index over devices
- \(W_k\) = Maximum strain energy in building with dampers at Target Displ.; 
  \(i = \) index over floor levels
Design Process for Velocity-Dependent Dampers using NSP (2)

\[ W_j = \frac{2\pi^2}{T_S} C_j \delta_{rj}^2 \]

Work done by j-th damper with building subjected to Target Displacement (assumes harmonic motion with amplitude equal to Target Displacement and frequency corresponding to Secant Stiffness at Target Displacement)

\[ \beta_{eff} = \beta + \frac{T_S \sum_j C_j \cos^2 \theta_j \phi_{rj}^2}{4\pi \sum_i m_i \phi_i^2} \]

Alternate expression for Effective Damping Ratio that uses modal amplitudes of first mode shape

Checking Building Component Behavior (Forces and Deformations)

For velocity-dependent dampers, must check component behavior at three stages:
1) Maximum Displacement
2) Maximum Velocity
3) Maximum Acceleration
2000 – Prestandard and Commentary for the Seismic Rehabilitation of Buildings (FEMA 356)

- Prestandard version of 1997 NEHRP Guidelines and Commentary for the Seismic Rehabilitation of Buildings (FEMA 273 & 274)

- Prepared by ASCE for FEMA

- Prestandard = Document has been accepted for use as the start of the formal standard development process (i.e., it is an initial draft for a consensus standard)
- Appendix to Chapter 13 entitled *Structures with Damping Systems*  
  (completely revised/updated version of 1994 and 1997 Provisions; Brief commentary provided)

- Intention:
  - Apply to all energy dissipation systems (EDS)
  - Provide design criteria compatible with conventional and enhanced seismic performance
  - Distinguish between design of members that are part of EDS and members that are independent of EDS.

The seismic force resisting system must comply with the requirements for the system’s Seismic Design Category, except that the damping system may be used to meet drift limits.

*No reduction in detailing is thereby allowed, even if analysis shows that the damping system is capable of producing significant reductions in ductility demand or damage.*
- Members that transmit damper forces to foundation designed to remain elastic

- Prototype tests on at least two full-size EDD’s (reduced-scale tests permitted for velocity-dependent dampers)

- Production testing of dampers prior to installation.

- Independent engineering panel for review of design and testing programs

- Residual mode concept introduced for linear static analysis. This mode, which is in addition to the fundamental mode, is used to account for the combined effects of higher modes. Higher mode interstory-velocities can be significant and thus are important for velocity-dependent dampers.
Methods of Analysis:

- Linear Static (Equivalent Lateral Force*)
  - OK for Preliminary Design
- Linear Dynamic (Modal Response Spectrum*)
  - OK for Preliminary Design
- Nonlinear Static (Pushover*)
  - May Produce Significant Errors
- Nonlinear Dynamic (Response History)
  - Required if $S_1 > 0.6 \, g$ and may be used in all other cases

*The Provisions allow final design using these procedures, but only under restricted circumstances.
Effective Damping Ratio
(used to determine factors, B, that reduce structure response)

$$\beta_m = \beta_I + \beta_{Vm} \sqrt{\mu} + \beta_H$$

- Hysteretic Damping Due to Post-Yield Behavior in Structure
- Equivalent Viscous Damping of EDS in the m-th Mode
- Inherent Damping Due to Pre-Yield Energy Dissipation of Structure
  ($\beta_I = 5\%$ or less unless higher values can be justified)
- Effective Damping Ratio in m-th mode of vibration
Equivalent Viscous Damping from EDS

\[ \beta_m = \beta_I + \beta_{Vm} \sqrt{\mu} + \beta_H \]

\[ \beta_{Vm} = \frac{\sum W_{mj}}{4\pi W_m} \]

Maximum Elastic Strain Energy of structure in m-th mode

\[ W_m = \frac{1}{2} \sum_{i} F_{im} \delta_{im} \]

Adjustment factor that accounts for dominance of post-yielding inelastic hysteretic energy dissipation
2000 - NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (6 of 8)
Part 1 – Provisions & Part 2 – Commentary (FEMA 368 & 369)

Base Shear Force

Minimum base shear for design of structure without EDS

\[ V_{min} = \max \left\{ \frac{V}{B_{V+I}} , 0.75V \right\} \]

Minimum base shear for design of seismic force resisting system

Spectral reduction factor based on the sum of viscous and inherent damping

To protect against damper system malfunction, maximum reduction in base shear over a conventional structure is 25%
V_{\text{min}} \geq 0.75V = \frac{V}{1.33}

Maximum base shear reduction factor

Maximum Added Damping WRT Minimum Base Shear
= 14 - 5 = 9%
Effect of Added Viscous Damping

Decreased Displacement
Decreased Shear Force (can not take full advantage of)